

THE
PROJECTION
OF THE
SPHERE,

ORTHOGRAPHIC, STEREOGRAPHIC
and GNOMONICAL.

Both Demonstrating the Principles, and Explaining
the Practice of these three several Sorts of Projection.

In Minimis Usus.



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U. S. N. O. J.

1. The first part of the document is a list of names and titles, including "The Hon. Mr. Justice" and "The Hon. Mr. Justice".

THE P R E F A C E.

THE Projection of the Sphere, or of its Circles has the same Relation to Spherical Trigonometry, that practical Geometry has to plane Trigonometry. For as one saves a deal of Calculation, by drawing a few right Lines, so does the other by drawing a few Circles. The Projection of the Sphere gives a Learner a good Idea of the Sphere and all its Circles, and of their several Positions to one another, and consequently of Spherical Triangles, and the Nature of Spherical Trigonometry.

I have here delivered the Principles of three sorts of Projection, in a small Compass; and yet the Reader will find here, all that is essential to the Subject, and yet nothing superfluous. For I think no more need be said, or indeed can be said about it, to make it intelligible and practicable. For here is laid down, not only the whole Theory, but the Practice likewise. Yet the practical Part is entirely disengaged from the Theory; so that any Body (tho' he has no Desire or Leisure to attain to the Theory,) may nevertheless, by help of the Problems, make himself Master of the Practice. For which End I have endeavoured to make all the Rules relating to Practice, plain, short, and easy, and at the same Time full and clear.

It is true the Solution of Problems this way, must be allowed to be imperfect; for there will always be some Errors in working, as well as in the Instruments we work with. But no Body in seeking an accurate Solution to a Problem, will trust to a Projection by Scale and Compass;
A 2 because

because this cannot be depended on in Cases of great Nicety. Yet where no great Exactness is required, it will be found very ready and useful; and besides, will serve to prove and confirm the Solution obtained by Calculation.

But then this Defect is abundantly recompens'd by the Easiness of this Method. For by Scale and Compass only, all Sorts of Problems belonging to the Sphere, as in Astronomy, Geography, Dialling, &c. may be solved with very little Trouble, which require a great deal of Time and Pains, to work out trigonometrically by the Tables. It likewise affords a great Pleasure to the Mind, that one can, in a little Time, describe the whole Furniture of Heaven and Earth, and represent them to the Eye, in a small Scheme of Paper.

But its principal Use is for such Persons (and that is by far the greater Number) as having no Opportunity for learning spherical Trigonometry, have yet a Desire to resolve some Problems of the Sphere. For such as these, this small Treatise will be of particular Service, because the practical Rules, especially of any one sort of Projection, may be learn'd in a very little Time, and are easily remembered. So that I have some Hopes I shall please all my Readers, whether theoretical or practical.

W. E.

THE

THE
PROJECTION
OF THE
SPHERE in PLANO.
DEFINITIONS.

1. **P**ROJECTION of the Sphere is the representing its Surface upon a Plane, call'd the *Plane of Projection*.

2. *Orthographic* Projection, is the drawing the Circles of the Sphere upon the Plane of some great Circle, by Lines perpendicular to that Plane, let fall from all the Points of the Circles to be projected.

3. The *Stereographic* Projection, is the drawing the Circles of the Sphere upon the Plane of one of its great Circles, by Lines drawn from the Pole of that great Circle to all the Points of the Circles to be projected.

4. The *Gnomonical* Projection, is the drawing the Circles of an Hemisphere, upon a Plane touching it in the Vertex, by Lines or Rays issuing from the Center of the Hemisphere, to all the Points of the Circles to be projected.

5. The *Primitive* Circle is that on whose Plane the Sphere is projected. And the Pole of this Circle is called the *Pole* of Projection. The Point from whence the *projecting* right Lines issue is the *projecting Point*.

6. The *Line of Measures* of any Circle is the common Intersection of the Plane of Projection and another Plane that passes thro' the Eye, and is perpendicular, both to the Plane of Projection and to the Plane of that Circle.

PROJECTION of the SPHERE, &c.

SCHOLIUM.

There are other Projections of the Sphere, as the *Cylindrical*, the *Scenographic* which belongs to Perspective, the *Globical* which belongs to Geography, *Mercators* for which see Navigation, &c.

AXIOM.

The *Place* of any visible Point of the Sphere upon the Plane of Projection, is where the projecting Line cuts that Plane.

Cor. If the Eye be apply'd to the projecting Point, it will view all the Circles of the Sphere, and every Part of them, in the Projection, just as they appear from thence in the Sphere itself.

Schol. The Projection of the Sphere is only the Shadow of the Circles of the Sphere upon the Plane of Projection, the Light being in the Place of the Eye or projecting Point.

The Signification of some Characters.

- + Added to.
- Subtracting the following Quantity.
- ∠ An Angle.
- = Equal to.
- ⊥ Perpendicular to.
- ∥ Parallel to.
- :: A Porportion.

THE
 ORTHOGRAPHIC PROJECTION
 OF THE
 SPHERE.

PROP. I.

IF a right Line AB is projected upon a Plane, it is **FIG.**
 projected into a right Line, and it's Length will be 3.
 to the Length of the Projection, as Radius to the
 Cosine of it's Inclination above that Plane.

For let fall the Perpendiculars Aa, Bb upon the Plane of Projection, then ab will be the Line it is projected into, but by Trigonometry AB is to Ao or $ab ::$ as Radius : to the Sine of B or Cosine of oAB .

Cor. 1. If a right Line is projected upon a Plane, parallel thereto, it is projected into a right Line parallel and equal to itself.

Cor. 2. If an Angle be projected upon a Plane which is parallel to the two Lines forming the Angle; it is projected into an Angle equal to itself.

Cor. 3. Any plane Figure projected upon a Plane parallel to itself, is projected into a Figure similar and equal to itself.

Cor. 4. Hence also the Area of any plane Figure, is to the Area of it's Projection :: as Radius to the Cosine of its Elevation or Inclination.

FIG.

P R O P. II.

A Circle perpendicular to the Plane of Projection is projected into a right Line equal to its Diameter.

For projecting Lines drawn through all the Points of the Circle fall in the common Section of the Planes of the Circle and of Projection, which is a right Line by *Eucl. XI. 3.* and equal to the Diameter of the Circle; because the Planes intersect in that Diameter. *Q. E. D.*

Cor. Hence any plane Figure, perpendicular to the Plane of Projection, is projected into a right Line. For the Perpendiculars from every Point, will all fall in the common Intersection of the Figure with the Plane of Projection.

P R O P. III.

1. *A Circle parallel to the Plane of Projection is projected into a Circle equal to itself, and concentric with the Primitive.*

Let BOD be the Circle, I its Center, C the Center of the Sphere, the Points I, B, O, D , are projected into the Points C, L, F, G , and therefore $OICF$, and $BICL$ are rectangled Parallelograms. Consequently $LC=BI=OI=FC$, by *Eucl. I. 34.* *Q. E. D.*

Cor. The Radius CL or CF is the Cosine of the Circle's Distance from the Primitive. For it is the Sine of AB .

P R O P. IV.

An inclin'd Circle is projected into an Ellipsis whose transverse Axis is the Diameter of the Circle.

2. Let $ADBH$ be the inclined Circle, P its Center; and let it be projected into $adbh$; draw the Plane $ABFCa$

of the SPHERE.

5

ABFCa through the Center C of the Sphere, perpendicular to the Plane of the given Circle and Plane of Projection, to intersect them in the Lines AB, ab; draw GPH, DE, perpendicular, and DQ parallel to AB; then because the Line GP, and the Plane of Projection are both perpendicular to the Plane ABF; therefore GH is parallel to the Plane of Projection, and therefore to gb. In the Circle ADB, $DQ^2 = GQH = gqb$, and $BP^2 = GP^2 = gp^2$. And (by *Eucl.* XI. 17.) $BP : EP$ or $DQ :: bp : ep$ or dq , and $BP^2 : DQ^2 :: bp^2 : dq^2$, that is $gp^2 : gqb :: bp^2 : dq^2$, and therefore agbb is an Ellipsis, whose Transverse gb is the Diameter of the Circle. Q. E. D.

FIG.
2.

Cor. 1. Since ab is perpendicular to gb, therefore ab is the conjugate Axis; and is twice the Sine of the $\angle ABb$ to the Radius gp. That is the conjugate Axis is equal to twice the Cosine of the Inclination, to the Radius of the Circle.

Cor. 2. The transverse Axis is equal to twice the Cosine of its Distance from its parallel great Circle. For $gb = GH = 2AP =$ twice the Sine of AK.

Cor. 3. The Extremities of the conjugate Axis are distant from the Center of the Primitive, by the Sines of the Circles nearest and greatest Distance from the Pole of the Primitive. Thus aC is the Sine of AN, and bC the Sine of BN.

Cor. 4. Hence also it is plain that the conjugate Axis always passes thro' the Center C of the Primitive, and is always in the Line of Measures of that Circle.

SCHOLIUM.

Every Circle in the Projection represents two equal Circles, parallel and equidistant from the Primitive. Every right Line represents two Semi-circles, one towards the Eye, the other in the opposite Side. Every Ellipsis represents two equal Circles, but contrarily inclin'd as AB, CD; one above the Primitive the other below it.

3.

FIG. And now the Theory being laid down, it remains only to deduce thence, some short Rules for Practice, as follows.

P R O P. V. P R O B.

5. *To project a Circle parallel to the Primitive.*

R U L E.

Take the Complement of its Distance from the Primitive, and set it from A to E; and with the Center C and Radius CD = perpendicular EF, describe the Circle Dg G.

By the plane Scale.

Take the Sine of its Distance from the Pole of the Primitive; with that Radius and the Center C describe the Circle.

P R O P. VI. P R O B.

4. *To project a right Circle, or one that is perpendicular to the Plane of Projection.*

R U L E.

Through the Center C of the Primitive, draw the Diameter AB, and take the Distance from its parallel great Circle, and set from A to E, and from B to D, and draw ED, the right Circle required.

By the Scale.

Take the Sine of the Circle's Distance from its parallel great Circle AB, and at that Distance draw a Parallel ED for the Circle required.

P R O P. VII. P R O B.

To project a given oblique Circle.

R U L E.

Draw the Line of Measures AB, and take the Circle's

cle's nearest Distance from the Primitive, and set from FIG. B to D upwards, if it be above the Primitive, or 6. downwards, if below; likewise take its greatest Distance, and set from A to E, and draw ED, and let fall the Perpendiculars EF, DG; and bisect FG in H, and erect the Perpendicular KHI, making $KH=HI=$ half ED, then describe an Ellipsis (by the Conic Sections) whose Transverse is IK and conjugate FG; and that shall represent the Circle given:

By the Scale.

Draw the Line of Measures AB; and take the Sines of the Circle's nearest and greatest Distance from the Pole of the Primitive, and set them from the Center C to F and G, (both ways if the Circle encompass the Pole, but the same way if it lye on one side the Pole,) bisect FG in H, and erect HK, HI perpendicular to FG, and equal to the Radius of the Circle given, or the Sine of its Distance from its own Pole; about the Axes FG, KI describe an Ellipsis, and 'tis done. 6.

SCHOLIUM.

An Ellipsis great or small may be described by Points, thus; thro' the Center D of the Circle and Ellipsis draw $BD \perp$ the transverse AR, and on AR erect a sufficient Number of Perpendiculars IK, *ik*, &c. and make as DB or $DA : DE :: IK : IF :: ik : if$, &c. then thro' all the Points E, F, *f*, &c. draw a Curve. 10.

PROP. VIII. PROB.

To find the Pole of a given Ellipsis.

RULE.

Thro' the Center of the Primitive C draw the Conjugate

FIG. jugate of the Ellipsis; on the extream Points F, G, erect the Perpendiculars F E, G D; or set the Transverse I K from E to D, and bissect E D in R, and let fall R P perpendicular to A B, then is P the Pole.

By the Scale.

7. Take C F, and C G, and apply to the Sines, and find the Degrees answering or the Supplements; then take the Sine of half the Sum of these Degrees, if F, G be both on one side of C, or the Sine of half the Difference, if they lie on contrary sides; and set it from C to the Pole P.

Or thus, Apply the Semi-transverse I H to the Sines, and set the Degrees from E to R.

P R O P. IX. P R O B.

To measure an Arch of a parallel Circle; or to set any Number of Degrees on it.

R U L E.

With the Radius of the Parallel, and one Foot in C describe a Circle G g, draw C G B, and C g b; then B b will measure the given Arch G g, or G g will contain the given Number of Degrees set from B to b. So that either being given finds the other.

P R O P. X. P R O B.

To measure any Part of a right Circle.

R U L E.

8. In the right Circle E D, let E A = A D, and let A B be to be measured: make C F = A E; with extent B A = F G describe the Arch G I; draw C G K to touch it in G; then is H K the Measure of A B. For F G = S, $\angle$ H C K to the Radius C F or A E, and B A is the same, by Cor. Pr. III.

Otherwise

Otherwise thus.

On the Diameter ED describe the Semi-circle END, draw AN, BO, LP perpendicular to ED, then ON is the Measure of BA, and NP of AL: and ON or NP may be measured as in Prop. IX.

Or thus by the Scale.

Let AL be measured, draw CD, and LM parallel to AC, then CM apply'd to the Sines gives the Degrees. For Radius CD : AD :: CM : AL.

Cor. If the right Circle pass thro' the Center, there is no more to do, but to raise Perpendiculars on it, which will cut the Primitive, as required. Or apply the Part of the right Circle to the Line of Sines.

PROP. XI. PROB.

To set off any Number of Degrees upon a right Circle, DE.

R U L E.

Draw CA ⊥ ED, and make the ∠HCK = the Degrees given, make CF = Radius AE, take FG the nearest Distance, and set from A to B: then AB = ∠HCK, the Degrees proposed.

Otherwise thus.

On ED describe the Semi-circle END, then by Prop. IX. set off NP = Degrees given, draw PL perpendicular to ED, then AL contains the Degrees required.

Or thus by the Scale.

Draw CD, take the given Degrees off the Sines, and set from C to M, and draw ML parallel to CA, then AL = Arch required.

PROP.

FIG.

P R O P. XII. P R O B.

To measure an Arch of an Ellipsis, or to set any Number of Degrees upon it.

R U L E.

9. About A R the transverse Axis of the Ellipsis describe a Circle A B R ; erect the Perpendiculars B E D, K F I, on A R ; then B K is the Measure of E F, or E F is the Representation of the Arch B K ; and B K is to be measured, or any Degrees set upon it, as in Prop. IX
- 10.

S C H O L I U M.

These Problems are all evident from the three first Propositions, and need no other Demonstration. If the Sphere be projected on any Plane parallel to the Primitive, the Projection will be the very same ; for being effected by parallel Lines, which are always at the same Distance, there will be produced the same Figure or Representation. Of all Orthographic Projections, those on the Meridian or on the Solstitial Colure commonly called the Analemma is most useful, because a great many of the Circles of the Sphere fall into right Lines or Circles ; whereas in the Projections upon other Planes, they are projected into Ellipses, which are hard to describe, which makes these Sorts of Projection to be neglected.

And by the same Rules that the Circles of the Sphere are projected upon a Plane, any other Figure may likewise be orthographically projected, by letting fall Perpendiculars upon the Plane from all the Angles, or all Points of the Figure, and joining these Points with right or curve Lines, as they are in the Figure itself.

THE

THE STEREOGRAPHIC PROJECTION OF THE SPHERE.

PROP. I.

ANY Circle passing thro' the projecting Point, is projected into a right Line.

For all Lines drawn from the projecting Point to this Circle, pass thro' the Intersection of this Circle and Plane of Projection, which is a right Line.

Cor. 1: A great Circle passing thro' the Poles of the Primitive is projected into a right Line passing thro' the Center. FIG. 12.

Cor. 2. Any Circle passing thro' the projecting Point is projected in a right Line perpendicular to the Line of Measures, and distant from the Center, the Semi-tangent of its nearest Distance from the Pole opposite to the projecting Point. Thus the Circle AE is projected into a right Line passing thro' G, and perpendicular to BC the Line of Measures, and GC is the Semi-tangent of EM.

PROP. II.

Every Circle (that passes not thro' the projecting Point) is projected into a Circle.

Case

FIG. *Case I.* Let the Circle EF be parallel to the Primitive BD ; Lines drawn to all Points of it from the projecting Point A , will form a Conic Surface, which being cut parallel to the Base by the Plane BD , the Section GH (into which EF is projected) will be a Circle by the Conic Sections.

11. *Case II.* Let BH be the Line of Measures to the Circle EF , draw FK parallel to BD , then Arch $AK = AF$, and therefore $\angle AFK$ or $AHG = AEF$; therefore in the Triangles AEF , AGH , the Angles at E and H are equal, and the Angle A common; therefore the Angles at F and G are equal. Therefore the Cone of Rays AEF (whose Base EF is a Circle) is cut by sub-contrary Section, by the Plane of Projection BD , and therefore, by the Conic Sections, the Section GH (which is the Projection of the Circle EF) will also be a Circle. *Q. E. D.*

P R O P. III.

12. *Any Point on the Sphere's Surface is projected into a Point, distant from the Center, the Semi tangent of its Distance from the Pole opposite to the projecting Point.*

Thus the Point E is projected into G , and F into H ; and CG is the semi-tangent of EM , and CH of MF .

Cor. A great Circle perpendicular to the Primitive is projected into a Line of semi-tangents passing thro' the Center. For MF is projected into CH its semi-tangent, and EM into the semi-tangent CG .

P R O P. IV.

The Angle made by two Circles on the Surface of the Sphere is equal to that made by their Representatives upon the Plane of Projection.

Let the Angle BPK be projected. Thro' the Angular point P and the Center C , draw the Plane of a great

great Circle PED perpendicular to the Plane of Projection EFG. Let a Plane PHG touch the Sphere in P; then since the Circle EPD is perpendicular both to this Plane and to the Plane of Projection, therefore it is perpendicular to their Interfection GH. The Angles made by Circles are the same as those made by their Tangents, therefore in the Plane PGH, draw the Tangents PH, PF, PG to the Arches, PB, PD, PK; and these will be projected into the Lines pH , pF , pK : Now I say the $\angle HPG = \angle HpG$. For the Angle $CPF =$ a right Angle $= C p A + C A p$; therefore taking away the equal Angles CPA and CAP, and $\angle p P F = C p A$ or $P p F$; consequently $p F = P F$. Therefore in the right-angled Triangles PFG and $p F G$, there are two Sides equal and the included $\angle$ right; therefore Hypothenuse $PG = p G$. And for the same Reason in the right-angled Triangles PFH and $p F H$, $PH = p H$. Lastly in the Triangles PHG and $p H G$, all the Sides are respectively equal, and therefore $\angle P = \angle p$. Q. E. D.

Cor. 1. The rumb Lines projected make the same Angles with the Meridians, as upon the Globe; and therefore are logarithmic Spirals on the Plane of the Equinoctial. For every Particle of the rumb coincides with some great Circle.

Cor. 2. The Angle made by two Circles on the Sphere is equal to the Angle made by the Radii of their Projections at the Point of Interfection. For the Angle made by two Circles on a Plane is the same with that made by their Radii drawn to the Point of Interfection.

PROP. V.

The Center of a projected (lesser) Circle perpendicular to the Primitive, is in the Line of Measures distant from the Center of the Primitive, the Secant of the lesser Circle's Distance from its own Pole; and its Radius is the Tangent of that Distance.

B

Let

FIG. 14. Let A be the projecting Point, EF the Circle to be projected, GH the projected Diameter. From the Centers C, D draw CF, DF, and the Triangles CFI, DFI are right angled at I; then $\angle IFC = \angle FCA = 2\angle FEA$ or $2\angle FEG = 2\angle FHG = \angle FDG$, therefore $\angle IFC + \angle IFD = \angle FDG + \angle IFD =$ a right Angle that is $\angle CFD$ is a right Angle, and the Line CD is the Secant of BF, and the Radius FD is the Tangent of it. *Q. E. D.*

Cor. If these Circles be actually described, 'tis Plain the Radius FD is a Tangent to the Primitive at F, where the lesser Circle cuts it.

P R O P. VI.

The Center of Projection of a great Circle is in the Line of Measures, distant from the Center of the Primitive, the Tangent of its Inclination to the Primitive; and its Radius is the Secant of its Inclination.

15. Let A be the projecting Point, EF the great Circle, GH the projected Diameter, D the Center; draw DA. The Angle EAF being in a Semi-circle is right. In the right-angled Triangle GAH, AC is perpendicular to GH, therefore $\angle GAC = \angle AHG$ and their double, $\angle ECB = \angle ADC$, and their Complements $\angle ECI = \angle CAD$. Therefore CD is the Tangent of ECI and Radius AD its Secant. *Q. E. D.*

16. *Cor. 1.* If the great oblique Circle AGBH be actually described upon the Primitive AIB. I say all great Circles passing thro' G will have the Centers of their Projections in the Line RS drawn thro' the Center D, perpendicular to the Line of Measures IH. For since all great Circles cut a semi-circle's Distance, all Circles passing thro' G must cut at the opposite Point H; and therefore their Centers must be in the Line RDS.

Cor. 2. Hence also if any oblique Circle GLH be required to make any given Angle with another BGAH, it will be projected the same way with regard

gard to GAH consider'd as a Primitive, and RS its Line of Measures; as the Circle BGA is on the Primitive BIA , and Line of Measures ID . And therefore the Tangent of the Angle AGL to the Radius GD , set from D to N , gives the Center of GL . For the $\angle NGD$ will then be equal to AGL , by Cor. 2. Prop. IV. and therefore GLH is rightly projected.

Cor. 3. And for the same Reason, if N be the Center of the Circle $GgHR$, the Centers of all Circles passing thro' g and R , will be in the Line rNs perpendicular to RS ; so n is the Center of grR . But then as g, R do not represent opposite Points of the Circle GgH , therefore all Circles passing thro' g, R , (as grR) will be lesser Circles, except $GgHR$.

P O R P. VII.

The projected Extremities of the Diameter of any Circle are in the Line of Measures, distant from the Center of the primitive Circle, the Semi-tangents of its nearest and greatest Distances from the Pole of Projection opposite to the projecting Point.

For the Diameter of the Circle EF is projected into GH , from the projecting Point A . But GC is the Semi-tangent of EB , and CH the Semi-tangent of BF . Q. E. D.

Cor. 1. The Points where an inclined great Circle cuts the Line of Measures, within and without the Primitive, is distant from the Center of the Primitive, the Tangent and Co-tangent of half the Complement of the Circle's Inclination to the Primitive.

For $CG =$ Tangent of half EB , or of half the Complement of IE the Inclination. And (because the $\angle EAF$ is right) CH is the Co-tangent of GAC or half EB .

Cor. 2. Hence the Center D of a projected Circle is in the Line of Measures distant, from the Center of

15.

17.

15.

17.

18.

FIG. the Primitive, half the Difference of the Semi-tangents of its nearest and greatest Distance from the opposite Pole, if it encompasses that Pole; but half the Sum of the Semi-tangents if it lie on one Side the Pole of Projection.

Cor. 3. And the Radius is half the Sum of the Semi-tangents if the Circle encompasses the Pole, or half the Difference if it lies on one Side.

17. Cor. 4. Hence also if p, q be the projected Poles, it will be $qG : pG :: qH : pH$.

For draw Gn parallel to qA , and since P, Q are the Poles, therefore qAp is a right Angle, and since the Angles GAp and pAH are equal, and Gn perpendicular to Ap , therefore $GA = An$; whence by similar Triangles $qG : qH :: An$ or $AG : AH :: Gp, pH$ (*Eucl. VI. 3.*); and consequently the Line qH is cut harmonically in the Points G, p .

P R O P. VIII.

The projected Poles of any Circle are in the Line of Measures, within and without the Primitive, and distant from its Center the Tangent and Co-tangent of half its Inclination to the Primitive.

19. The Poles P, p of the Circle EF are projected into D and d ; and CD is the Tangent of CAD or half BCP , that is of half GCI the Inclination of the Circle ICK parallel to EF . Likewise Cd is the Tangent of CAd , or the Co tangent of CAD . *Q. E. D.*

Cor. 1. The Pole of the Primitive is its Center; and the Pole of a right Circle is in the Primitive.

19. Cor. 2. The projected Center of any Circle is always between the projected Pole (nearest to it on the Sphere) and the Center of the Primitive; and the projected Centers of all Circles lie between the projected Poles. For the middle Point of EF or its Center is projected into S ; and all the Points in Pp (in which are all the Centers) are projected into Dd .

Cor.

Cor. 3. If P be the projected Center of any Circle EFG , any right Lines EG , FH passing thro' P will intercept equal Arches EF , GH . For in any Circle of the Sphere, any two Lines, passing thro' the Center, intercept equal Arches; and these are projected into right Lines, passing thro' the projected Center P , and therefore EF , GH , represent equal Arches. FIG. 20.

P R O P. IX.

If $EFGH$, $efgb$ represent two equal Circles, whereof EFG is as far distant from its Pole P , as efg is from the projecting Point. I say any two right Lines (eEP , and fFP ,) being drawn thro' P , will intercept equal Arches (in Representation) of these Circles; on the same Side, if P falls within the Circles; but on the contrary Side, if without: That is $EF=ef$, and $GH=gb$. 20.
21.

For by the Nature of the Section of a Sphere; any two Circles passing thro' two given Points or Poles on the Surface of the Sphere, will intercept equal Arches of two other Circles equi-distant from these Poles. Therefore the Circles EFG and efg on the Sphere are equally cut by the Places of any two Circles passing thro' the projecting Point and the Pole P , on the Sphere. But these Circles (by Prop. I.) are projected into the right Lines Pe and Pf , passing thro' P ; and the intercepted Arches representing equal Arches on the Sphere, are therefore equal, that is $EF=ef$, and $GH=gb$.

Cor. 1. If a Circle is projected into a right Line EF , perpendicular to the Line of Measures EG ; and if from the Center C a Circle efP be described passing thro' its Pole P , and Pf be drawn; then Arch $ef=EF$. And if any other Circle be described whose vertex is P , the Arch ef will always be equal to EF . 22.

Cor. 2. Hence also if from the Pole of a great Circle there be drawn two right Lines, the intercepted Arch of the projected great Circle will be equal to the intercepted Arch of the Primitive.

FIG. Cor. 3. After the same manner, if there be two
 23. equal Circles EF , ef , whereof one is as far from the
 24. Pole P as the other is from the Pole of Projection C ,
 opposite to the projecting Point. Then any Circle
 drawn thro' the Points P , C , will intercept equal
 Arches $Ff = ef$; and $GH = gh$, between it and the
 Line of Measures PCG . For this is true on the
 Sphere and their Projections are the same.

Cor. 4. If from an angular Point be drawn two right
 Lines thro' the Poles of its Sides; the intercepted
 Arch of the Primitive, will be equal to that Angle,
 for the Distance of the Poles is equal to that Angle.

PROP. X.

25. If QH , NK be two equal Circles, whereof NK is
 26. as far from the projecting Point as QH from its Pole P :
 and if they be projected into the Circles whose Radii are
 MC or CL and DF or FG , F being the Center of
 DG , and E the projected Pole. I say the Pole E will be
 distant from their Centers in Proportion to the Radii of
 the Circles; that is, $CE : EF :: CL : DF$ or FG .

For since NK and ML are parallel, and Arch
 $NI = PH$, therefore $\angle ELI = NKI$ (or nKI)
 $= GIP$. Therefore the Triangles IEL and IEG
 are similar, whence $EL : EI :: EI : EG$. Again
 the Angle $EMI = KNI = PIQ$, and therefore the
 Triangles IEM and IED are similar, whence $EM :$
 $EI :: EI : ED$. Therefore $EF = EL \times EG = EM$
 $\times ED$. Consequently $EM : EL :: EG : ED$; and
 by Composition $\frac{EM+EL}{2} : \frac{EM-EL}{2} :: \frac{EG+ED}{2} :$
 $\frac{EG-ED}{2}$, that is $CM : EC :: FG : EF$. Q.E.D.

25. Cor. 1. Hence if the Circle KN be as far from the
 26. projecting Point, as QH is from either of its Poles,
 and if E , O , be its projected Poles; then will $EL :$
 $EM :: ED :: EG :: OD : OG$.

This

of the SPHERE.

19

This follows from the foregoing Demonstration, and Cor 4. Prop. VII. FIG.

Cor. 2. Hence also if F be the Center and FD the Radius of Projection of any Circle QH, and E, O the projected Poles, then $EF : DF :: DF : FO$. 25. 26.

For it follows from Cor. 1. that $\frac{EG+ED}{2} : \frac{EG-ED}{2} :: \frac{OG+OD}{2} : \frac{OG-OD}{2}$.

Cor. 3. Hence if the Circle DBG, be as far from its projected Pole P, as LMN is from the projecting Point; and if any right Lines be drawn thro' P, as MPG, NPK, they will cut off similar Arches GK, MN in the two Circles. 27. 28.

For from the Centers C, F, draw the Lines CN, FK, then since the Angles CPN, and FPK are equal, and by this Prop. $CP : CN :: FP : FK$; therefore (Eucl. VI. 7.) the Triangles PCN and PFK are similar; and the Angle $PCN = \angle PFK$; therefore the Arches MN and GK are similar.

Cor. 4. Hence also if thro' the projected Pole P of any Circle DBG, a right Line BPK be drawn. Then I say the Degrees in the Arch GK shall be the Measure of DB in the Projection. And the Degrees in DB shall be the Measure of GK in the Projection. 27. 28.

For (by Prop. IX) the Arch MN is the Measure of DB, and therefore GK which is similar to MN, will also be the Measure of it.

Cor. 5. The Centers of all projected Circles are all beyond the projected Poles, (in respect to the Center of the Primitive); and none of their Centers can fall between them.

Cor. 6. Hence it follows (by Cor. 5. and Prop. VIII. Cor. 3.) that all Circles that are not parallel to the Primitive have equal Arches on the Sphere represented by unequal Arches on the Plane of Projection. For if P be the projected Center, then GH is greater than EF. 20.

Stereographic PROJECTION

S C H O L I U M.

It will be easy by the foregoing Propositions to describe the Representation of any Circle, and the Reverse will easily show what Circle of the Sphere any projected Circle represents. What follows hereafter is deduced from the foregoing Propositions, and will easily be understood without any other Demonstration.

If the Sphere was to be projected on any Plane parallel to the Primitive, 'tis all the same thing. For the Cones of Rays issuing from the projecting Point, are all cut by parallel Planes into similar Sections, it only makes the Projections bigger or less, according to the Distance of the Plane of Projection, whilst they are still similar; and amounts to no more than projecting from different Scales upon the same Plane. And therefore the projecting the Sphere on the Plane of a lesser Circle is only projecting it upon the great Circle parallel thereto, and continuing all the Lines of the Scheme to that lesser Circle.

P R O P. XI. P R O B.

To draw a Circle parallel to the Primitive, at a given Distance from its Pole.

R U L E.

29. Thro' the Center C draw two Diameters AB, DE, perpendicular to one another. Take in your Compasses the Distance of the Circle from the Pole of the Primitive opposite to the projecting Point, and set it from D to F, from E draw EF to intersect AB in I; with the Radius CI and Center C describe the Circle GI required.

By the plane Scale.

With the Radius CI, equal to the Semi-tangent of the Circle's Distance from the Pole of Projection opposite to the projecting Point, describe the Circle IG. Where the Radius of Projection CA is the Tangent of 45° , or the Semi-tangent of 90° .

P R O P.

PROP. XII. PROB.

FIG.

To draw a lesser Circle perpendicular to the Primitive, at a given Distance from the Pole of that Circle.

R U L E.

Thro' the Pole B draw the Line of Measures A B, make BG the Circle's Distance from its Pole, and draw CG, and GF perpendicular to it; with the Radius FG describe the Circle GI required.

30.

By the Scale.

Set the Secant of the Circle's Distance from its Pole from C to F, gives the Center. With the Tangent of that Distance for a Radius describe the Circle GI.

Or thus, Make BG the Circle's Distance from its Pole, and GF its Tangent set from G gives F the Center, thro' G describe the Circle GI.

Cor. Hence a great Circle perpendicular to the Primitive, is a right Line CDE drawn thro' the Center perpendicular to the Line of Measures.

SCHOLIUM.

When the Center F lyes at too great a Distance, draw EG, to cut AB in H; or lay the Semi-tangent of DG from C to H. And thro' the three Points G, H, I, draw a Circle with a Bow.

PROP. XIII. PROB.

To describe an oblique Circle at a given Distance from a Pole given.

R U L E.

Draw the Line of Measures AB thro' the given Point *p*, if that Point is given; and draw DE $\perp$ to it, also draw EpP. Or if the Point *p* is not given, set the Height of the Pole above the Primitive from B to P. Then from P set off PH = PI = Circle's Distance

31.

FIG. Distance from its Pole; and draw EH , EI , to intersect AB in F and G . About the Diameter FG describe the Circle required.

31.

By the Scale.

If the Point p is given, apply Cp to the Semi-tangents and it gives the Distance of the Pole from D , the Pole of Projection opposite to the projecting Point. This Distance being had, you'll easily find the greatest and nearest Distances of the Circle from the Pole of the Primitive opposite to the projecting Point: Take the Semi-tangents of these Distances and set from C to G and F , both the same way if the Circle lie all on one Side, but each its own way, if on different Sides of D . And then FG is the Diameter of the Circle required to be drawn.

Cor. 1. If F be the Pole of a great Circle as of DLE . Draw EFH , and make $HP = DH$, and draw EpP , and then p is its Center.

Cor. 2. Hence it will be easy to draw one Circle parallel to another.

PROP. XIV. PROB.

Thro' two given Points A, B, to draw a great Circle.

R U L E.

32. Thro' one of the Points A , draw a Line thro' the Center, ACG ; and EF perpendicular to it. Then draw AE , and EG perpendicular to it. Thro' the three Points A , B , G draw the Circle required.

Or thus, from E (found as before), draw EH , and then HCI , and lastly EIG , gives G a third Point thro' which the Circle must pass.

By the Scale.

Draw ACG ; and apply AC to the Semi-tangents, find the Degrees, let the Semi-tangent of its Supplement from C to G , a third Point.

Or

Or thus; Apply AC to the Tangents, and set the Tangent of its Complement from C to G. And thro' the three Points A B G, describe the Circle required. FIG. 32.

For since HEI or AEG is a right Angle, therefore A, G are opposite Points of the Sphere; and therefore all Circles passing thro' A and G are great Circles.

SCHOLIUM.

If the Points A, B, G lie nearly in a right Line, then you may draw a Circle thro' them with a Bow.

PROP. XV. PROB.

About a Pole given, to describe a Circle thro' a given Point.

RULE.

Let P be the Pole, and B the given Point: Thro' P, B describe the great Circle AD (by Prop. XIV.) whose Center is E; thro' the Center C draw CPH; and from the Center E, draw EB, and BF perpendicular to it. To the Center F, and Radius FB describe the Circle BGH required. 33.

PROP. XVI. PROB.

To find the Poles of any Circle FNG.

RULE.

Thro' its Center draw the Line of Measures AG, and DE perpendicular to it. Draw EFH, and set its Distance (from its own Pole) from H to P, and draw EpP, then p is the Pole. 31.

Or thus, draw EFH, EIG, and bisect HI in P, and draw EpP, and p is the internal Pole. Lastly draw PCQ, and EQq, and q is the external Pole.

In a great Circle DLE, draw ELK, and make DH=AK, (or KH=AD), and draw EFH, and F is the Pole.

By the Scale.

Apply CF to the Semi-tangents, and note the Degrees.

FIG. grees. Take the Sum of these Degrees and of the
 31. Circle's Distance from its Pole, if the Circle lie all on one Side, but their Difference if it incompass the Pole of Projection; set the Semi-tangent of this Sum or Difference from C to the internal Pole p . And the Semi-tangent of its Supplement Cq , gives the external Pole q .

Or thus; Apply CF and CG to the Semi-tangents, set the Semi-tangent of half the Sum of the Degrees (if the Circle lies all one way) or of half the Difference (if it encompasses the Pole of Projection), from C to the Pole p ; and the Semi-tangent of the Supplement, Cq , gives the external Pole q .

In a great Circle as DLE, draw the Line of Measures AB, perpendicular to DE, and set the Tangent and Co-tangent of half its Inclination, from the Center C, different ways to F and f , gives the internal and external Poles F, f .

P R O P. XVII. P R O B.

To draw a great Circle at any given Inclination above the Primitive; or making any given $\angle$ with it, at a given Point.

R U L E.

34. Draw the Line of Measures AB; and DCE perpendicular to it. Make $EK = 2HD =$ twice the Complement of the Circle's Inclination, (or $DK = 2AH =$ twice the Inclination); and draw EKF, then F is the Center of EGD, the Circle required.

By the Scale.

Set the Tangent of the Inclination in the Line of Measures from C to E, then F is the Center. Set the Semi-tangent of the Complement from C to G; then GF or DF is the Radius.

Or the Secant of the Inclination set from G or D to F gives the Center.

Cor.

Cor. To draw an oblique Circle to make a given Angle with a given oblique Circle DGE at D. Draw EGH, and set the given Angle from H to I, and draw ELI. Thro' D, L, E describe a great Circle. FIG. 34.

PROP. XVIII. PROB.

Thro' a given Point P to draw a great Circle, to make a given Angle with the Primitive.

RULE.

Thro' the Point given P and the Center C draw the Line AB; and DE perpendicular to it. Set the given Angle from A to H and from H to K, and draw BGK; with Radius CG, and Center C describe the Circle GIF; and with Radius BG and Center P cross that Circle in F. Then with Radius FP and Center F, describe the Circle LPM required. 35.

By the Scale.

With the Tangent of the given Angle and one Foot in C, describe the Arch FG. With the Secant of the given Angle and one Foot in the given Point P, cross that Arch at F. From the Center F describe a Circle thro' the Point P.

PROP. XIX. PROB.

To draw a great Circle to make a given Angle with a given oblique Circle EPR, at a given Point P, in that Circle.

RULE.

Thro' the Center C and the given Point P, draw the right Line DE; and AB perpendicular to it, draw APG, and make $BM = 2 DG$, and draw AM to cut DE in I. Draw IQ perpendicular to DE, then IQ is the Line wherein the Centers of all Circles are found which pass thro' the Point P. Find N the Center of the given Circle FPR, and make the Angle 36.

FIG. 36. gle NPL equal to the given Angle, then L is the Center of the Circle HPK required.

By the Scale.

Thro' P and C draw DE ; apply CP to the Semi-tangents, and set the Tangent of its Complement from C to I (or the Secant from P to I) on DI erect the Perpendicular IQ . Find the Center N of FPR , and make the Angle $NPL =$ Angle given, and L is the Center.

Cor. If one Circle is to be drawn perpendicular to another, it must be drawn thro' its Poles.

PROP. XX. PROB.

To draw a great Circle thro' a given Point P , to make a given Angle with a given great Circle DE .

RULE.

37. About the given Point P as a Pole describe (*a*) the great Circle FG ; find I the Pole of the given Circle DE (*b*), and about the Pole I (*c*) describe the small Circle HKL at a Distance equal to the given Angle, to intersect FG in H ; about the Pole H describe (*c*) the great Circle APB required.

PROP. XXI. PROB.

To draw a great Circle to cut two given great Circles abd , ebf at given Angles.

RULE.

50. Find the Pole, s , r , of the two given Circles, by Prop. 16. about which draw two Parallels pbk , pnk at the Distances respectively equal to the Angles given, by Prop. 13. the Point of Intersection p , is the Pole of the Circle moq required.

Cor. Hence, to draw a right Circle to make with an oblique Circle abd , any given Angle. Draw a Parallel pbk at a Distance from the Pole of the oblique Circle, equal to the given Angle. Its Intersection

(*a*) By Cor. 1. Pr. 13. (*b*) By Pr. 16. (*c*) By Pr. 13.

tion *f* with the Primitive gives the Pole of the right FIG. Circle required *g c t*.

PRO P. XXII. PROB.

To lay any Number of Degrees on a great Circle, or to measure any Arch of it.

R U L E.

Find the internal Pole *P* of the given Circle *DEH* (by Prop. 16.); lay the Degrees on the Primitive from *A* to *F*, and draw *PA*, *PF*, intercepting the Part required *DE*. Or to measure *DE*, draw *PEF* and *PDA*, and *AF* is its Measure, and apply'd to the Line of Chords shows how many Degrees it is.

38.

Or thus; Find the external Pole *p* of the given Circle, set the given Degrees from *I* to *K*, and draw *pI*, *pK*, intercepting the Part *DE* required. Or to measure *DE*, thro' *D* and *E* draw *pI*, *pK*, then *KI* is the Measure of *DE*.

Or thus; Thro' the internal Pole *P*, draw the Lines *DPG* and *EPL*; seting the given Degrees from *G* to *L* in the Circle *GL*, then *DE* is the Arch required. Or if *DE* be to be measured, then the (equal) Degrees in the Arch *GL* is equal to *DE*.

Or thus; Set the given Degrees from *G* to *H* (in Circle *GL*) and from the external Pole *p*, draw *pG*, *pH*, intercepting *DE* the Arch required. Or to measure *DE*, draw *pDG*, *pEH*, then the Degrees in *GH*, is equal to *DE*.

By the Scale for right Circles.

Let *CA* be the right Circle, take the Number of Degrees off the Semi-tangents and set from *C* to *D* for the Arch *CD*. Or if the given Degrees are to be set from *A*, then take the Degrees off the Semi-tangents from 90° towards the beginning, and set from *A* to *D*. And if *CD* was to be measured, apply it to the beginning of the Semi-tangents. And to measure *AD*, apply it from 90° backwards, and the Degrees intercepted gives its Measure.

38.

SCHO-

FIG.

S C H O L I U M.

The Primitive is measured by the Line of Chords, or else it is actually divided into Degrees.

P R O P. XXIII. P R O B.

To set any Number of Degree, on a lesser Circles or to measure any Arch of it.

R U L E.

38.

Let the lesser Circle be DEH ; find its internal Pole P , by Prop. 16. describe the Circle AFK parallel to the Primitive, by Prop. 11. and as far from the projecting Point, as the given Circle DE is from its internal Pole P , set the given Degrees from A to F , and draw PA , PF intersecting the given Circle in D , E ; then DE is the Arch required. Or to measure DE , draw PDA , PEF , and AF shows the Degrees in DE .

Or thus; Find the external Pole p , of the given Circle, by Prop. 16. describe the lesser Circle AFK as far from the projecting Point, as DE the given Circle is from its Pole p , by Prop. 11. Set the Degrees from I to K and draw pDI , pEK , then DE represents the given Number of Degrees. Or to measure DE ; draw pDI , pEK ; and KI is the measure of DE .

Or thus; Let O be the Center of the given Circle DEH ; thro' the internal Pole P , draw Lines DPG , EPL , divide the Quadrant GQ into 90 equal Degrees, and if the given Degrees be set from G to L , then DE will represent these Degrees. Or the Degrees in GL will measure DE .

Or thus; Divide the Quadrant GR into 90 equal Parts or Degrees, and set the given Degrees from G to H , and draw pDG , pEH , from the external Pole p . Then DE will represent the given Degrees. Or thro' D , E drawing pDG , pEH , then the Number of equal Degrees in GH is the Measure of DE .

SCHOLIUM.

Any Circle parallel to the Primitive is divided or measured, by drawing Lines from the Center to the like Divisions of the Primitive. Or by help of the Chords on the Sector set to the Radius of that Circle.

PROP. XXIV. PROB.

The Measure any Angle.

RULE.

By Cor. 1. Pr. 13. about the angular Point as a Pole, describe a great Circle, and note where it intersects the Legs of the Angle, thro' these Points of Intersection, and the angular Point, draw two right Lines, to cut the Primitive, the Arch of the Primitive intercepted between them is the Measure of the Angle. This needs no Example.

Or thus; by Prop. 16. Find the two Poles of the containing Sides, (the nearest, if it be an acute Angle, otherwise the furthest) and thro' the angular Point and these Poles, draw right Lines to the Primitive; then the intercepted Arch of the Primitive is the Angle required.

SCHOLIUM.

Because in the Stereographic Projection of the Sphere, all Circles are projected either into Circles or right Lines, which are easily described; therefore this sort of Projection is prefer'd before all others. Also those Planes are prefer'd before others to project upon, where most Circles are projected into right Lines, they being easier to describe and measure than Circles are; such are the Projections on the Planes of the Meridian and solstitial Colure.

THE GNOMONICAL PROJECTION

OF THE SPHERE.

PROP. I.

FIG.
39.

EVERY great Circle as BAD is projected into a right Line perpendicular to the Line of Measures, and distant from the Center the Co-tangent of its Inclination, or the Tangent of its nearest Distance from the Pole of Projection.

Let $CBED$ be perpendicular both to the given Circle BAD and Plane of Projection, and then the Interfection CF will be the Line of Measures. Now since the Plane of the Circle BD and the Plane of Projection are both perpendicular to $BCDE$, therefore their common Section will also be perpendicular to $BCDE$, and consequently to the Line of Measures CF . Now since the projecting Point A is in the Plane of this Circle, all the Points of it will be projected into that Section, that is into a right Line passing thro' d , and perpendicular to Cd . And Cd is the Tangent of CD , or Cotangent of CdA .
Q. E. D.

Cor. 1. A great Circle perpendicular to the Plane of Projection is projected into a right Line passing thro' the Center of Projection; and any Arch is projected

jected into its correspondent Tangent. Thus the Arch CD is projected into the Tangent Cd. FIG. 39.

Cor. 2. Any Point as D or the Pole of any Circle, is projected into a Point *d* distant from the Pole of Projection C, the Tangent of that Distance.

Cor. 3. If two great Circles be perpendicular to each other, and one of them passes thro' the Pole of Projection; they will be projected into two right Lines perpendicular to each other.

For the Representation of that Circle which passes thro' the Pole of Projection is the Line of Measures of the other Circle.

Cor. 4. And hence if a great Circle be perpendicular to several other great Circles, and its Representation pass thro' the Center of Projection; then all these Circles will be represented by Lines parallel to one another, and perpendicular to the Line of Measures or Representation of that first Circle.

P R O P. II.

If two great Circles intersect in the Pole of Projection; their Representations shall make an Angle at the Center of the Plane of Projection equal to the Angle made by these Circles on the Sphere.

For since both these Circles are perpendicular to the Plane of Projection; the Angle made by their Intersections with this Plane, is the same as the Angle made by these Circles. Q. E. D.

P R O P. III.

Any lesser Circle parallel to the Plane of Projection is projected into a Circle, whose Center is the Pole of Projection, and Radius the Tangent of the Circle's Distance from the Pole of Projection.

Let the Circle PI be parallel to the Plane GF, then the equal Arches PC, CI are projected into the equal
C 2 Tangents

FIG. Tangents GC , CH ; and therefore C the Point of Contact and Pole of the Circle PI and of the Projection, is the Center of the Representation GH . *Q. E. D.*

39.

Cor. If a Circle be parallel to the Plane of Projection and 45 Degrees from the Pole; it is projected into a Circle equal to a great Circle of the Sphere; and may therefore be look'd upon as the primitive Circle in this Projection, and its Radius the Radius of Projection.

P R O P. IV.

40.

Every lesser Circle (not parallel to the Plane of Projection) is projected into a Conic Section, whose transverse Axis is in the Line of Measures, and whose nearest Vertex is distant from the Center of the Plane the Tangent of its nearest Distance from the Pole of Projection; and the other Vertex is distant the Tangent of its furthest Distance.

Let BE be parallel to the Line of Measures dp , then any Circle is the Base of a Cone whose Vertex is at A , and therefore that Cone being produced will be cut by the Plane of Projection in some Conic Section. Thus the Circle whose Diameter is DF will be cut by the Plane in an Ellipsis whose Transverse is df ; and Cd is the Tangent of CAD , and Cf of CF . In like manner the Cone $A FE$ being cut by the Plane, f will be the nearest Vertex; and the other Point into which E is projected is at an infinite Distance. Also the Cone $A FG$ (whose Base is the Circle FG) being cut by the Plane, f is the nearest Vertex; and GA being produced gives d the other Vertex. *Q. E. D.*

Cor. 1. If the Distance of the furthest Point of the Circle be less than 90° from the Pole of Projection, then it will be projected into an Ellipsis.

Thus DF is projected into df , and DC being less than 90° , the Section df is an Ellipsis whose Vertices are at d and f ; for the Plane df cuts both Sides of the Cone dA , fA .

Cor.

Cor. 2. If the furthest Point be more than 90° Degrees from the Pole of Projection, it will be projected into an Hyperbola. Thus the Circle FG is projected into an Hyperbola whose Vertices are f and d , and transverse fd . For the Plane dp cuts only the side Af of the Cone.

Cor. 3. And in the Circle FE , where the furthest Point E is 90° from C ; it will be projected into a Parabola, whose Vertex is f . For the Plane dp (cutting the Cone FAE) is parallel to the Side AE .

Cor. 4. If H be the Center, and K, k, l the Focus of the Ellipsis, Hyperbola, or Parabola; then $HK = \frac{Ad - Af}{2}$ for the Ellipsis, and $Hk = \frac{Ad + Af}{2}$ for the Hyperbola; and (drawing fn perpendicular on AE) $fl = \frac{nE + Ff}{2}$, for the Parabola; which are the Representations of the Circles DF, FG, FE respectively. All this appears from the Conic Sections.

PROP. V.

Let the Plane TW be perpendicular to the Plane of Projection TU . And BCD a great Circle of the Sphere in the Plane TW . And let the great Circle BED be projected in the right Line bek . Draw $CQS \perp bk$, and $Cm \parallel$ to it and equal to CA , and make $QS = Qm$; then I say any Angle $QSt = Qt$.

Suppose the Hypotenuse AQ to be drawn, then since the Plane ACQ is perpendicular to the Plane TU , and bQ is $\perp$ to the Intersection CQ , therefore bQ is perpendicular to the Plane ACQ , and consequently bQ is perpendicular to the Hypotenuse AQ . But $AQ = Qm = QS$, and QS is also perpendicular to bQ . Therefore all Angles made at S cut the Line bQ in the same Points as the Angles made at A ; but by the Angles at A the Circle BED is projected into the Line bQ . Therefore the Angles at S are the

FIG. Measures of the Parts of the projected Circle bQ ;

41. and S is the *dividing Center* thereof. *Q. E. D.*

Cor. 1. Any great Circle tQb is projected into a Line of Tangents to the Radius SQ . For Qt is the Tangent of the Angle QSt to the Radius QS or Qm .

Cor. 2. If the Circle bC pass thro' the Center of Projection; then A the projecting Point is the *dividing Center* thereof. And Cb is the Tangent of its correspondent Arch CB , to CA the Radius of Projection.

P R O P. VI.

41. *Let the parallel Circle GEH be as far from the Pole of Projection C as the Circle FKI is from its Pole P ; and let the Distance of the Poles C, P be bisected by the Radius AO , and draw bAD perpendicular to AO . Then any right Line bek draw thro' b , will cut off the Arches $bl = Fn$, and $ge = kf$, (supposing f the other Vertex), in the Representations of these equal Circles in the Plane of Projection.*

For let $G, E, R, L, H, N, R, K, I$ be respectively projected into the Points $g, e, r, l, b, n, r, k, f$. Then since in the Sphere, the Arch $BF = DH$, and Arch $BG = DI$. And the great Circle $BEKD$ makes the Angles at B and D equal, and is projected into a right Line as bl . Therefore the triangular Figures BFN and DHL are similar, and equal, and likewise BGE , and DIK are similar and equal, and $LH = NF$, and $KI = EG$. Whence it is evident their Projections $lb = nF$, and $kf = ge$. *Q. E. D.*

P R O P. VII.

42. *If blg and Fnk be the Projections of two equal Circles, whereof one is as far from its Pole P as the other from its Pole C , which is the Center of Projection; and if the Distance of the projected Poles C, p be divided in o , so that the Degrees in Co, op , be equal, and the Perpendicular oS be erected to the Line of Measures gb . I say the Lines pn, Cl , drawn from the Poles C, p thro' any*

Point Q in the Line oS , will cut off the Arch $Fn = \text{FIG.}$
 $\angle QCP$. 42.

For drawing the great Circle GPI in a plane Perpendicular to the Plane of Projection. The great Circle AO perpendicular to CP is projected into oS by Prop. I. Cor. 3. Now let Q be the Projection of q , and since pQ , CQ are right Lines, therefore they represent the great Circles Pq , Cq . But the spherical Triangle PqC is an Isocles-triangle, and therefore the Angles at P and C are equal. But because P is the Pole of FI , therefore, the great Circle Pq continu'd, will cut an Arch off $FI = \angle CPq = \angle PCq = \angle QCP$ by Prop. II. That is (since Fn represents the Part cut off from FI) Arch $Fn = \text{Arch } lb$ or $\angle QCb$. Q. E. D.

Cor. Hence if from the projected Pole p of any Circle, a Perpendicular be erected to the Line of Measures; it will cut off a Quadrant from the Representation of that Circle. For that Perpendicular will be parallel to oS , Q being at an infinite Distance.

P R O P. VIII.

Let Fnk be the Projection of any Circle FI , and p the projected Pole P . And if Cg be the Co-tangent of CAP and gB perpendicular to the Line of Measures gC , and CAP be bisected by AO , and the Line OB , be drawn to any Point B , and also pB cutting Fnk in d . I say the Angle $gOB = \text{Arch } Fd$. 42.

For the Arch PG is a Quadrant, and the $\angle goA = \angle gpa + \angle oAp =$ (because GCA and gAp are right Angles) $gAC + oAp = gAC + CAo = \angle gAo$. Therefore $gA = go$, consequently o is the dividing Center of gB the Representation of GA ; and consequently by Prop. V. $\angle goB$ is the Measure of gB . But since pg represents a Quadrant, therefore p is the Pole of gB , and therefore the great Circle pdB passing thro' the Pole of the Circles gB and Fn will cut off

FIG. off equal Arches in both, that is $Fd = gB = \angle goB$.
 42. Q. E. D.

Cor. The $\angle goB$ is the Measure of the Angle gpB . For the Triangle gpB represents a Triangle on the Sphere wherein the Arch which gB represents is equal to the Angle which $\angle p$ represents, because gp is 90 Degrees. Therefore goB is the Measure of both.

SCHOLIUM.

Thus far I have treated of the Theory; what follows is the practical Part, and depends altogether on what is above delivered, in which I think no Difficulty can occur. In the Gnomonical Projection, the Plane projected on, is supposed to touch the Hemisphere to be projected, in its Vertex; and the Point of Contact will be the *Center of Projection*. But if it be required to project upon any Plane parallel to this touching Plane, the Process will be no way different, and is only taking a greater or lesser Radius of Projection, according to the greater or lesser Distance; which is in effect projecting a greater or lesser Sphere upon its touching Plane.

When you have the Sphere to project this way, upon a given Plane; it will assist the Imagination, if you suppose yourself placed in the Center of the Sphere, with your Face towards the Plane, whose Position is given; and from thence projecting with your Eye, the Circles of the Sphere upon this Plane.

PROP. IX. PROB.

To draw a great Circle, thro' a given Point, and at a given Distance from the Pole of Projection.

RULE.

43. Describe the Circle ADB with the Radius of Projection, and thro' the given Point P draw the right Line PCA , and CE perpendicular to it; make the Angle $CAE =$ given Distance of the Circle from C , and thro' E describe the Circle EFG , and thro' P draw the Line PK touching the Circle in I , then is IK the Circle required. By

By the plain Scale.

With the Tangent of the Circle's Distance from the Pole of Projection C, describe the Circle EIF, and draw PK to touch this Circle.

PROP. X. PROB.

To draw a great Circle perpendicular to a given great Circle which passes thro' the Pole of Projection; and at a given Distance from that Pole. 43.

RULE.

Draw the Primitive ADB. Let CI be the given Circle, draw CL perpendicular to CI, and make the Angle CLI = the given Distance; thro' I draw KP parallel to CL for the Circle required.

By the Scale.

In the given Circle CI, set the Tangent of the given Distance, from C to I; thro' I draw KP perpendicular to CI, then KP is the Circle required.

PROP. XI. PROB.

To measure any Part of a great Circle; or to set any Number of Degrees thereon.

RULE.

Let EP be the great Circle; thro' C draw ID perpendicular to EP, and CB parallel to it. Let EBD be a Circle described with the Radius of Projection CB, make IA = IB; then A is the dividing Center of EP, consequently drawing AP, the $\angle IAP$ = Measure of the given Arch IP. Or if the Degrees be given, make the $\angle IAP$ = these given Degrees, which cuts off IP the Arch correspondent thereto. 44.

By the Scale.

Draw ICD perpendicular to EP; apply CI to the Tangents, and set the Semi-tangent of its Complement from C to A, gives the dividing Center of EP, &c.

PROP.

FIG.

PROP. XII. PROB.

To draw a great Circle to make a given Angle with a given great Circle, at a given Point; or to measure an Angle made by two great Circles.

R U L E.

51. Let P be the given Point, and PB the given great Circle. Draw thro' P , and C the Center of Projection, the Line PCG , to which from C draw CA perpendicular, and equal to the Radius of Projection. Draw PA and AG perpendicular to it, at G erect BD perpendicular to GC , cutting PB in B ; draw AO bisecting the Angle CAP ; then at the Point O , make $BOD =$ Angle given, and from D draw the Line DP , then BPD is the Angle required.

Or if the Degrees in the Angle BPD be required, from the Points B, D draw the Lines BO, DO ; and the Angle BOD is the measure of BPD ,

Cor. If an Angle be required to be made at the Pole or Center of Projection, equal to a given Angle; this is no more than drawing two Lines from the Center making the Angle required. And if one great Circle be to be drawn $\perp$ to another great Circle, it must be drawn thro' its Pole.

PROP. XIII. PROB.

To project a lesser Circle parallel to the Primitive.

R U L E.

43. With the Radius of Projection AC , and Center C , describe the primitive Circle ADB , by *Cor. Pr. III.* and draw ACB , and GCE perpendicular to it. Set the Circle's Distance from its Pole from B to H , and from H to D , and draw AED . With Radius CE describe the Circle EFG required.

By the Scale.

With the Radius CE equal to the Tangent of the Circle's

Circle's Distance from its Pole, describe the Circle FIG. EFG, for the Circle required.

PROP. XIV. PROB.

To draw a lesser Circle perpendicular to the Plane of Projection.

RULE.

Thro' the Center of Projection C, draw its parallel great Circle TI. At C make the Angle ICN and TCO = the given Circle's Distance from its parallel great Circle TI; make CL = Radius of Projection, and draw LM perpendicular to CL. Set LM from C to V, or CM from C to F. Then thro' the Vertex V, between the Assymptotes CN, CO describe the Hyperbola WVK. Or to the Focus F, and Semi-transverse CV, describe the Hyperbola; for the Circle required.

48.

Otherwise by Points.

Thro' the Center of Projection C draw the Line of Measures CF, and TCI perpendicular to it, draw any Number of right Lines CV, DE, GH, IR, &c. and PQ, RS, TW, &c. perpendicular to TI. And by Prop. XI. make CV, DE, GH, &c. each equal to the Distance of the given Circle from its parallel great Circle; then all the Points W, S, Q, V, E, H, K, &c. join'd by a regular Curve will be the Representation of the Circle required.

Or thus.

Make the Angle iak = Distance of the given Circle from its parallel great Circle. Then thro' the Center of Projection C, draw the great Circle TCI parallel to the Circle given, upon which erect the perpendicular CA = Radius of Projection. Also draw any Number of right Lines CV, DE, GH, IK, &c. perpendicular to TI. Then take each of the Distances from A to C, D, G, I, &c. and set them from

FIG. 48. from a to $c, g, d, i, \&c.$ and to ai draw the Perpendiculars $cv, de, gh, ik, \&c.$ and make $CV, DE, GH, IK, \&c.$ respectively equal to $cv, de, gh, ik, \&c.$ which gives the Points $V, E, H, K, \&c.$ after the same manner on the other Side find the Points $Q, S, W, \&c.$ then thro' all these Points $W, S, Q, V, E, H, K, \&c.$ draw a regular Curve, which will be an Hyperbola representing the Circle given.

By the Scale.

Take the Tangent of the Circle's Distance from its parallel great Circle, and set it from C (the Center of Projection) to V , and the Secant thereof from C to F . Then with the Semi-transverse CV , and Focus F , describe the Hyperbola $WVHK$.

P R O P. XV. P R O B.

To project any lesser oblique Circle given.

R U L E.

45. Draw the Line of Measures dp , and at C the Center of Projection draw $CA \perp$ to dp and $=$ Radius of Projection; with the Center A , describe the Circle $DCFG$; and draw RAE parallel to dp . Then take the greatest and least Distances of the Circle from the Pole of Projection and set from C to D and F , for the Circle DF ; and from A the projecting Point draw Aff , and Ad , then df will be the transverse Axis of the Ellipsis. But if D fall beyond the Line RE , as at G , then draw a Line from G backward thro' A to d , and then df is the Transverse of an Hyperbola. But if the Point D fall in the Line RE as at E , then the Line AE no where meets the Line of Measures, and the Projection of E is at an infinite Distance, and then the Circle will be projected into a Parabola whose Vertex is f . Lastly, bisect df in H the Center, and for the Ellipsis take half the Difference of the Lines Ad, Af and set from H to K for the Focus. But for the Hyperbola take half the

the Sum of $A d$, $A f$, and set from H to the Focus k of the Hyperbola. Then with the Transverse $d f$ and Focus K or k describe the Ellipsis $d M f$, or the Hyperbola $f m$ for the Projection of the Circle given. FIG. 45.

But for the Parabola make $E Q = F f$, and draw $f n \perp A Q$, and set $\frac{1}{2} n Q$ from f to k the Focus. Then with the Vertex f and Focus k describe the Parabola $f m$, for the Projection of the given Circle $F E$.

Otherwise by Points.

Thro' the Center of Projection C , draw the Line of Measures $C F$, passing thro' the Pole P (if P is given; but if not, find it, by setting off $C P =$ the Distance of that Pole, from the Center of Projection, by Pr. XI.) then set off $P D$, $P F$ equal to the given Distance from its Pole, by Prop. XI. Thro P draw a sufficient Number of right Lines, $L \lambda$, $M \mu$, $N n$, $O o$, $R r$, $S s$, &c. which will all represent great Circles. Find the dividing Centers of each of these Lines; and by Prop. XI. set off upon each of them from P , the given Distance of the Circle from its Pole as $P L$, $P \lambda$, $P M$, $P \mu$, &c. and thro' all the Points L , M , D , O , R , &c. draw a Curve Line, for the Circle required. 49.

Or thus.

Draw the Line of Measures $P C G$, and by Prob. XI. make $C G =$ the Distance of the Parallel great Circle from the Pole of Projection, and draw $A G K$ perpendicular to it, which will represent a great Circle whose Pole is P . Draw any Number of right Lines thro' P to $A K$, as $A P$, $B P$, $H P$, &c. and by Prop. XI. set off from $A K$, the Parts $A L$, $B M$, $H O$, &c. each equal to the Circle's Distance from its parallel great Circle. Then all the Points L , M , D , O , &c. being join'd by a regular Curve, will represent the parallel Circle required.

Or thus.

Thro' the Center of Projection C draw the Line of Measures

FIG. Measures DCF, and the Radius of Projection CW

42. perpendicular to it, and $AGK \perp GC$, for a great Circle whose Pole is P. Draw $wp = WP$, and $wa \perp$ to it, draw any Number of right Lines, AP, BP, GP, &c. and make pg, pb, pa &c. = PG, PB, PA, &c. also make the $\angle pwl$ and $pw\kappa$ = the Circle's Distance from its Pole P (or awl = the Distance from its parallel great Circle); and upon PG, PB, PA, &c. make PD, PM, PL &c. = pd, pm, pl , &c. respectively. Or make GD, BM, AL, &c. = gd, bm, al , &c. After the same manner, find the Points O, R, &c. and thro' all the Points R, O, D, M, L, &c. draw a regular Curve making no Angles, which will represent the parallel required. Likewise where any Line ap cuts $w\kappa$, that Distance from p will give the Point λ , or is = $P\lambda$ and so of any other of the Lines bp, gp , &c.

The Reason of this Process will be plain, if you suppose the Points p, w apply'd to P, W; and g, b, a , &c. successively to G, B, A, &c. for then d, m, l , will fall upon D, M, L, &c.

By the Scale.

45. Take the Tangents of the Circles nearest and furthest Distance from the Pole of Projection and set from C to f and d , gives the Vertices, and bisect df in H; then take half the Difference, or half the Sum, of the Secants of the greatest and least Distances from the Pole of Projection, and set from H, to K or k for the Focus of the Ellipsis or Hyperbola, which may then be described.

49. *Cor.* If the Curve be required to pass thro' a given Point S; measure PS by Prop. XI. and then the Curve may be drawn by this Problem.

PROP.

PROP. XVI. PROB.

FIG.

To find the Pole of any Circle in the Projection, DMF.

47.

RULE.

From the Center of Projection C, draw the Radius of Projection CA perpendicular to the Line of Measures DF. And to A the projecting Point, draw DA, FA, and bisect the Angle DAF by the Line AP, then P is the Pole. But if the Curve be an Hyperbola, as *f m*, Fig. 45. you must produce *d A*, and bisect the Angle *f A G*. And in a Parabola, where the Point *d* is at an infinite Distance, bisect the Angle *f A E*.

45.

Or thus; Drawing CA perpendicular to DC, draw DA, and make the Angle DAP = the Circle's Distance from its Pole, gives the Pole P.

47.

By the Scale.

Draw the Radius of Projection CA $\perp$ to the Line of Measures DF. Apply CD, CF to the Tangents, and set the Tangent of half the Difference of their Degrees from C to P, if D, F lie on contrary Sides of C; but half the Sum if on the same side, gives P the Pole.

Or thus; by Prop. XI, set off, from D to P, the Circle's Distance from its Pole, gives the Pole P.

Cor. If it be a great Circle as BG; draw the Line of Measures GC, and CA $\perp$ to it, and equal to the Radius of Projection. Make GAP a right Angle, and P is the Pole.

46.

PROP. XVII. PROB.

To measure any Arch of a lesser Circle; or to set any Number of Degrees thereon.

RULE.

Let F π be the given Circle. From the Center of Projection C draw CA perpendicular to the Line of Measures

FIG. 46. Measures GH. To P the Pole of the given Circle draw AP, and AO bisecting the Angle CAP. And draw AD perpendicular to AO. Describe the Circle G/H (by Prop. XIII.) as far from the Pole of Projection C, as the given Circle is from its Pole P. And thro' any given Point n in the Circle F n , draw D n l, gives Hl the Number of Degrees = F n . Or the Degrees being given and set from H to l, the Line Dl cuts off F n equal thereto.

Or thus; AO being drawn as before, erect OS perpendicular to CO; thro' the given Point n draw P n cutting OS in Q, then thro' Q draw Cl, and the Angle QCP is = F n . Or making QCP = the Degrees given, draw PQ n , and Arch F n = these Degrees.

Or thus; AO, AP being drawn as before, draw AG perpendicular to AP, and GB perpendicular to GC. Thro' the given Point n draw PB cutting GB in B, and draw OB, then the $\angle GOB$ = Arch F n . Or making $\angle GOB$ = the given Degrees; draw PB, and it cuts off F n = the Degrees given.

By the Scale.

Let C be the Center of Projection, P the Pole of the given Circle. Apply CP to the Tangents, and set the Tangent of its half from C to O, and the Co-tangent of its half from C to D; with Radius CG = Tangent of the Degrees in FP the given Circle's Distance from it's Pole, describe the Circle GSH. Then Dl drawn thro' n or l cuts off Hl = F n .

Or thus; O being found as before, erect OS perpendicular to CO. Thro' the given Point n , draw PQ n , and $\angle QCH$ = F n .

Or thus; Apply CP to the Tangents. and set the Co-tangent thereof from C to G, erect GB perpendicular to GC. Thro' n draw P n B, and draw BO, then $\angle GOB$ = F n .

Cor. If the lesser Circle be perpendicular to the Plane of Projection as V H K. You have no more to do but to draw the Perpendiculars V C, H G to its parallel great Circle C I. Then C G (measured by Prop. XI.) will be equal to V H; or the Degrees set from C to G, cuts of V H equal thereto. FIG. 48.

S C H O L I U M.

This Sort of Projection is little used by reason several of the Circles of the Sphere fall in Ellipses and Hyperbola's which are very difficult to describe. Notwithstanding it is very convenient for solving some Problems of the Sphere, because all the great Circles are projected into right Lines. And this Sort, or the Gnomonic Projection, is the very Foundation of all Dialling. For if the Sphere be projected on any Plane, and upon that Side of it on which the Sun is to shine; and the projected Pole be made the Center of the Dial, and the Axis of the Globe the Stile or Gnomon, and the Radius of Projection its Height; you will have a Dial drawn with all its Furniture. Upon this Account it deserves to be more taken Notice on than at present it is. I have in the foregoing Propositions, given I think, all the fundamental Principles of this kind of Projection, having met with little or nothing done upon this Subject before.

G E N E R A L P R O B L E M.

To project the Sphere upon any given Plane.

Before you can project the Sphere upon any Plane you must have a perfect Knowledge of all its Circles and their Positions upon the Globe; the Distances of all lesser Circles from their Poles, and from their parallel great Circles. The Angles made by great Circles, or their Inclinations to one another, particularly to the primitive Circle on whose Plane (or a parallel thereto) you are about to project the Sphere.

D

Then

FIG. Then after the primitive Circle is described, you must describe all Circles that are concern'd in the Problem, according to the Rules of that Sort of Projection you are going to use, and the Intersections of these Circles will determine the Problem.

The principal Points, Angles and Circles of the Sphere are as follows.

I. *Points.*

52. 1. Zenith is the Point over our Heads, Z.
53. 2. Nadir is the Point under our Feet N.
55. 3. Poles of the World are 2 Points round which the diurnal Revolution is performed, P the North Pole p the South Pole.
4. The Center of the Earth or of the Heavens, C.
5. Equinoctial Points, are the Points of Intersection of the Equator and Ecliptic, γ , π .
6. Solstitial Points, are the beginning of Cancer and Capricorn ϕ , ψ .

II. *Great Circles.*

1. Equinoctial, is a Circle 90 Degrees distant from the Poles of the World, as E Q.
2. Meridians, are Circles passing thro' the Poles of the World as P $\odot$ p, P E p, &c.
3. Solstitial Colure, is a Meridian passing thro' the Solstitial Points, as P ϕ p.
4. Equinoctial Colure, is a Meridian passing thro' the Equinoctial Points, P C p.
5. Ecliptic is the Circle thro' which the Sun seems to move in a Year, ϕ , ψ . it cuts the Equinoctial at an Angle of $23^{\circ} 30'$, passing thro' the Equinoctial Points. In this are reckon'd the 12 Sines γ , δ , π , ϕ , η , θ , ι , κ , λ , μ , ν , ξ .
6. Horizon, is a Circle dividing the upper from the lower Hemisphere, H O, 90° distant from the Zenith and Nadir.
7. Azimuths, are Circles passing thro' the Zenith and Nadir, Z $\odot$ N.
8. Circles

8. Circles of Longitude in the Heavens, pass thro' FIG. the Poles of the Ecliptic and cut it at right Angles. 52.
 9. Meridian of a Place, is that Meridian which 53.
 passes thro' the Zenith, as P Z H. 55.

III. *Lesser Circles.*

1. Parallels of Latitude are parallel to the Equinoctial on the Earth, Parallels of Altitude are parallel to the Horizon, Parallels of Declination are parallel to the Equinoctial in the Heavens.
 2. Tropics, are 2 Circles distant $23^{\circ} 30'$ from the Equinoctial, the Tropic of Cancer towards the North, the Tropic of Capricorn towards the South.
 3. Polar Circles, are distant $23^{\circ} 30'$ from the Poles of the World, the arctic Circle towards the North, the Antarctic towards the South.

IV. *Angles and Arches of Circles.*

1. Sun's (or Star's) Altitude is an Arch of the Azimuth between the Sun and Horizon, as $\odot B$.
 2. Amplitude is an Arch of the Horizon, between Sun rising and the East, or Sun setting and the West.
 3. Azimuth, is an Arch of the Horizon between the Sun's Azimuth Circle, and the North or South, as H B, or O B. Or it is the Angle at the Zenith, H Z B or O Z B.
 4. Right Ascension is an Arch of the Equator between the Sun's Meridian, and the first Point of Aries, as ΥK .
 5. Ascensional Difference is an Arch of the Equinoctial between the Sun's Meridian, and that Point of the Equinoctial that rises with him, or it is the Angle at the Pole between the Sun's and the Six o'Clock Meridian.
 6. Oblique Ascension, is the Sum or Difference of the right Ascension and the Ascensional Difference.
 7. Sun's Longitude, is an Arch of the Ecliptic, between the Sun and first Point of Aries, as $\Upsilon \odot$.

- FIG. 8. Declination, is an Arch of the Meridian, between the Equinoctial and the Sun, as $\odot K$.
52. 9. Latitude of a Star, is an Arch of a Circle of Longitude between the Star and Ecliptic.
53. 10. Latitude of a Place is an Arch of the Meridian between the Equinoctial and the Place.
55. 11. Diff. of Longitude on the Earth, is an Arch of the Equator between the Meridians of the two Places; or 'tis the $\angle$ of the Pole between these Meridians.
12. Hour of the Day is an Arch of the Equinoctial between the Meridian of the Place and the Sun's Meridian, as $E K$, or it's the Angle they make at the Pole as $E P \odot$.

E X A M P. I.

To project the Sphere upon the Plane of the Meridian, for May 1, 1747. Lat. $54\frac{1}{2}$. at a Quarter past 9 o' Clock before Noon.

I. By the Orthographic Projection.

52. With the Chord of 60° describe the primitive Circle or Meridian of the Place $H Z O N$; thro' the Center C draw the Horizon $H O$; set the Lat. $54\frac{1}{2}$ from O to P and from H to p , and draw $P p$ the 6 o'Clock Meridian. Thro' C draw $E Q$ perpendicular to $P p$ for the Equinoctial. Make $E D$, $Q d$ $18^\circ 5'$ the Declination *May 1st* and draw $D d$ the Sun's Parallel for that Day. By Prop. XI. make $\odot G$ ($3\frac{1}{4}$ Hours or) $48^\circ 45'$ the Sun's Distance from the Hour of 6, then $\odot$ is the Sun's Plane. Thro' $\odot$, by Pr. V. draw $A L$ parallel to $H O$ for the Sun's parallel of Altitude. By Prop. VII. draw the Meridian $P \odot p$ and the Azimuth $Z \odot N$. Also the Ecliptic will be an Ellipsis passing thro' $\odot$, which cannot conveniently be drawn in this Projection. Also draw the Parallel $S s$ 18° below the Horizon, and where it intersects $D d$ is the Point of Day-break, if there is any. Now the Sun is at d at 12 o'Clock at Night, and rises at

at R, at 6 o'Clock is at G, due East at F, at $\odot$ a FIG. Quarter past 9, and is at D in the Meridian at 12 52. o'Clock.

Draw GI parallel to HO. Then GR measured by Prop. X. is $27^{\circ} 14'$, and turn'd into Time (allowing 15 Deg. for an Hour) shows how long the Sun rises before six to be $1^h 49^m$. GI measured by Prop. X. gives the Azimuth at six $79^{\circ} 16'$. CR by Cor. Pr. X. gives the Amplitude $32^{\circ} 19'$, and CF gives his Altitude when East $22^{\circ} 25'$. FG $13^{\circ} 28'$ (turn'd into time) shows 54^m or how long after six he is due East. IO is his Altitude at six $14^{\circ} 38'$. AH $41^{\circ} 53'$ is his Altitude at $\odot$ or a Quarter past 9; and $\odot$ L measured by Pr. X. is his Azimuth from the North at the same time, $122^{\circ} 40'$. And thus the Place of the Moon or a Star being given, it may be put into the Projection, as at *. And its Altitude, Azimuth, Amplitude, Time of rising, &c. may all be found, as before for the sun.

II. Stereographically.

To project the Sphere on the Plane of the Meridian, the projecting Point in the western Point of the Horizon. With Cord of 60, draw the primitive Circle HZON, and thro' C draw HO for the Horizon, and ZN perpendicular thereto for the prime Vertical. Set the Latitude from O to P and from H to p , and draw Pp the six o'Clock Meridian, and EQ perpendicular thereto for the Equinoctial. Make ED, Qd the Declination, and by Prop. XII. draw DGd the Sun's Parallel for the Day. Draw the Meridian, P $\odot$ p by Pr. XVII. making an Angle of $41^{\circ} 15'$ with the Primitive, to intersect the Sun's parallel in $\odot$ the Sun's Place at $9^h \frac{1}{4}$. Thro' $\odot$, by Prop. XII. draw the Parallel of Altitude A $\odot$ L. Thro' $\odot$ draw, by Pr. XVII. the Azimuth Z $\odot$ N. And by Pr. XII. draw the Parallel Ssd 18° below the Horizon, if it cut Rd, gives the Point of Day break. And thro' G draw the Parallel of Altitude GI. Lastly by Prop. XX.

FIG. XX. thro' $\odot$ draw the great Circle $\gamma \odot \simeq$ cutting the Equinoctial EQ at an Angle of $23^\circ 30'$, and this is the Ecliptic, γ the first Point of Aries, and $\simeq$ that of Libra.

53.

This done dR measur'd by Pr. XXIII. is $62^\circ 46'$, shows the Time of Sun rising. CR by Pr. XXII. is the Amplitude $32^\circ 19'$. GI $79^\circ 16'$ by Pr. XXIII. the Sun's Azimuth at fix. IO $14^\circ 38'$ his Altitude at fix. CF $22^\circ 25'$ by Pr. XXII. his Altitude when East. GF $13^\circ 28'$ the Time when he is due East. $\odot B$ $41^\circ 53'$ by Pr. XXII. his Altitude at a Quarter past 9. The $\angle \odot ZP$ $122^\circ 40'$ by Pr. XXIV. his Azimuth at that Time. Also $\gamma \odot$, by Pr. XXII. is his Longitude $51^\circ 7'$. γK his right Ascension, $48^\circ 40'$.

And the Place of the Moon or a Star being given, it may be put into the Scheme as at $*$; and its Time of Rising, Amplitude, Azimuth, &c. found as before.

III. Gnomonically.

54.

To project the Eastern Hemisphere upon a Plane parallel to the Meridian. About the Center of Projection C describe the Circle HON with tangent of 45 the Radius of Projection, for the Primitive. Thro' C draw the Horizon HO . And the Prime vertical ZN perpendicular thereto. Set the Lat. $54\frac{1}{2}$ from H to a , and draw the 6 o'Clock Meridian Pp , and the Equinoctial EQ perpendicular to it. Set the Tangent of $48^\circ 45'$ (equal to $3\frac{1}{4}$ Hours) from C to E , and by Pr. X, draw the Meridian EL parallel to Pp . Make $Ee = Ea$, and $\angle Ee\odot = 18^\circ 5'$ the Sun's Declination, then by Pr. XI. $\odot$ is the Sun's Place. Thro' $\odot$ draw the Hyperbola $D\odot d$ (by Prop. XIV.) for the Sun's Parallel of Declination. And draw $\odot B$ perpendicular to HO , for his Azimuth Circle. And draw GI perpendicular to HO , and $RM, FT \parallel Pp$. Also the Ecliptic is a right Line passing thro' $\odot$ and cutting EQ at an Angle of $23^\circ 30'$ which is difficult to draw in this Projection. Also by Pr. XIV. draw the Parallel Ss 18° below the Ho-

rizon, and if it intersects Dd , it gives the Point of FIG. 54.
 Sun rise. Then if by Prop. XVII. or XI. you measure GR , or rather CM , $27^{\circ} 14'$, you have the Time of Sun-rising; GF or CT $13^{\circ} 28'$, the Time when he is due East. Also by Prop. XI. if you measure CR you have the Amplitude $32^{\circ} 19'$. CI the Complement of his Azimuth at six, $10^{\circ} 44'$. IG by Pr. XII. his Altitude at six $14^{\circ} 38'$. CF his Altitude when East, $22^{\circ} 25'$. And by Pr. XI. $\odot B = 41^{\circ} 53'$, his Altitude a Quarter past 9. CB the Complement of his Azimuth at that Time, $32^{\circ} 40'$.

And the Place of the Moon or a Star being given, its Place in the Projection may be determined as before, and all the Requisites found.

E X A M P. II.

To project the Sphere upon the Plane of the solstitial Colure for Lat. $54\frac{1}{2}$. May 12th 1747. at 10 o'Clock in the Morning.

Stereographically.

The Projection of the western Hemisphere, the first Point of Libra the projecting Point. Describe the solstitial Colure $PEpQ$, and the equinoctial Colure Pp perpendicular to it; and thro' C draw the Equinoctial EQ perpendicular to Pp . Set $23^{\circ} 30'$ from E to $\S$ and from Q to $\wp$ and draw the Ecliptic $\S\wp$. Set the Sun's Longitude $61^{\circ} 42'$ from C to $\odot$, and thro' $\odot$ draw $P\odot Kp$ for the 10 o'Clock Meridian. Make KA (two Hours or) 30° and draw PAp the Meridian of the Place. Set the Latitude of the Place $54\frac{1}{2}$ from A to Z , and Z is the Zenith. About the Pole Z describe the great Circle BHS for the Horizon of the Place. Thro' Z and $\odot$ draw an Azimuth Circle $Z\odot B$. 55.

Then you have $\odot K$ the Sun's Declination $20^{\circ} 33'$. CK his right Ascension $59^{\circ} 35'$. $\odot B$ his Altitude at 10 o'Clock $49^{\circ} 10'$. the $\angle AZ\odot$ or $PZ\odot$ his Azimuth at 10= HB , $45^{\circ} 44'$. H the South Point of the Horizon. I the Point of the Ecliptic that is in the Meridian. T the Point of the Ecliptic that is setting in the Horizon.

E X -

E X A M P. III.

*To project the Sphere on the Plane of the Horizon,
Lat. $35\frac{1}{2}$, N. July 20th 1747. at 10 o'Clock.*

Gnomonically.

56.

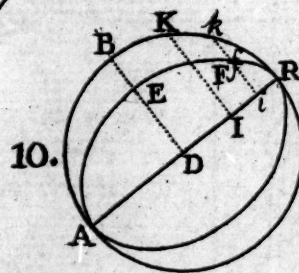
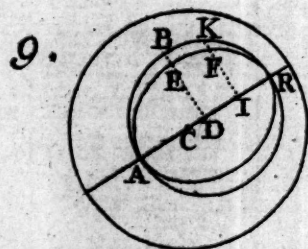
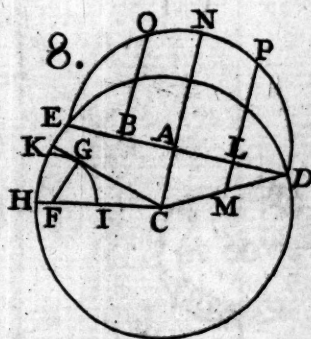
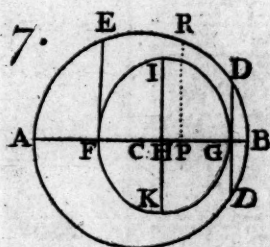
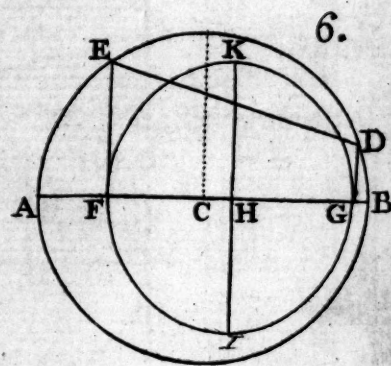
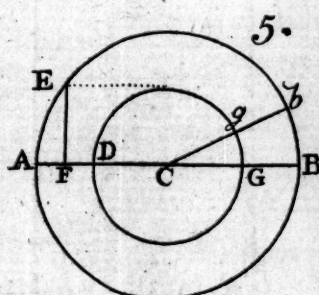
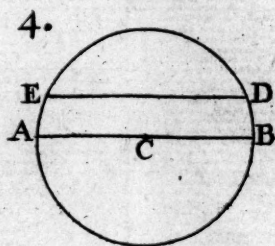
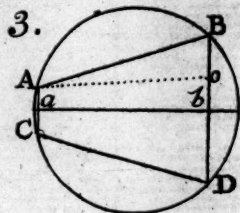
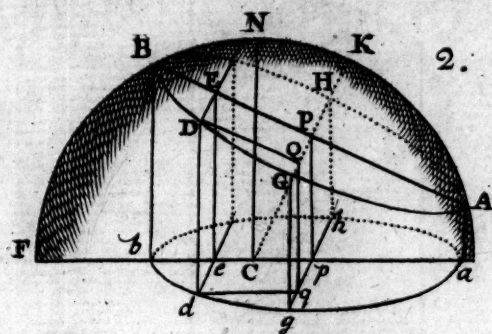
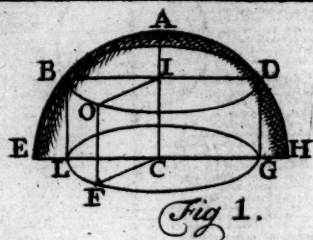
To project the upper Hemisphere on a Plane parallel to the Horizon. With the Radius of Projection and Center C, describe the primitive Circle ADB Thro' C draw the Meridian PE, and AS perpendicular to it for the prime Vertical. Set off CP $35\frac{1}{2}$ the Lat. and P is the N. Pole, and perpendicular to CP draw Pp the six o'Clock Meridian. Set the Complement of the Lat. from C to E. And draw EQ perpendicular to EC for the Equinoctial. Make EB 30° (or 2 Hours) and draw the 10 o'Clock Meridian PB. Set the Sun's Declination $18^\circ 27'$ from B to $\odot$. And $\odot$ is the Place of the Sun at 10 o'Clock. Thro' $\odot$ draw the Azimuth Circle CQ. Likewise thro' $\odot$ a parallel to the Equinoctial EQ may easily be described by Pr. XV. for the Sun's Parallel that Day.

Then C $\odot$ measured by Prop. XI. is $31^\circ 30'$ the Complement of the Altitude. And the Angle EC $\odot$ measured by Cor. Prop. XII. is his Azimuth, $65^\circ 10'$.

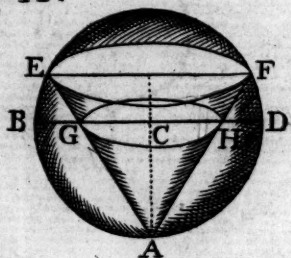
Cor. After this manner may any Problems of the Sphere be solved by any of these Projections, or upon any Planes, but upon some more commodiously than upon others. And if in a spherical Triangle any sides or Angles be required, they may be projected according to what is given therein, according to any of these Kinds of Projection before delivered; and it will be most easily done, when you chuse such a Plane to project on, that some given Side may be in the Primitive, or a given Angle at the Center: And then you need draw no more Lines or Circles than what are immediately concerned in that Problem. But always chuse such a Plane to project on, where the Lines and Circles are most easily drawn, and so that none of them run out of the Scheme.

F I N I S.

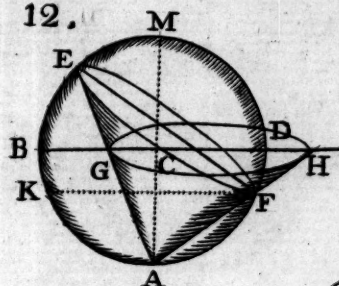




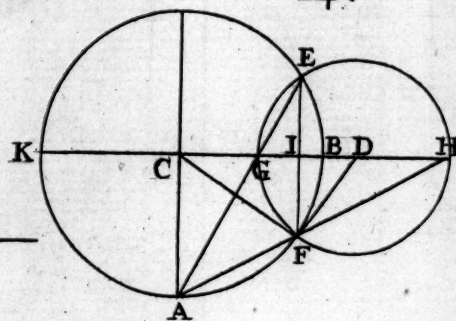
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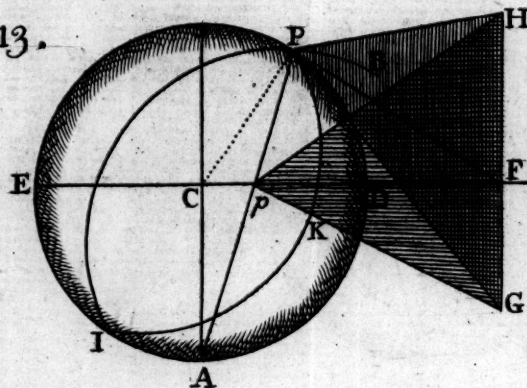
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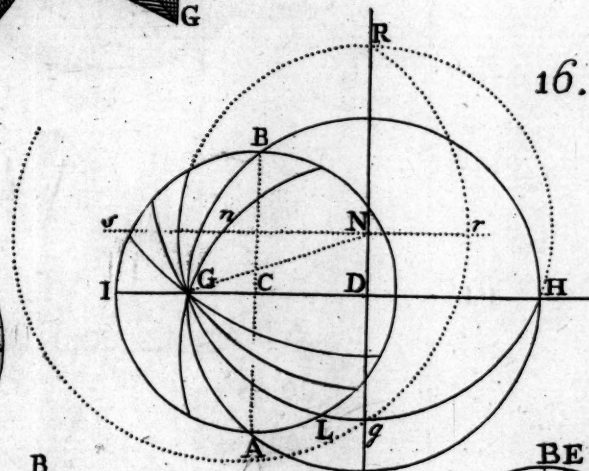
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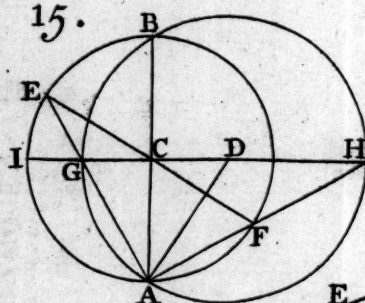
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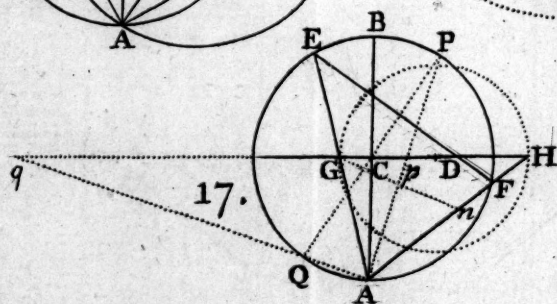


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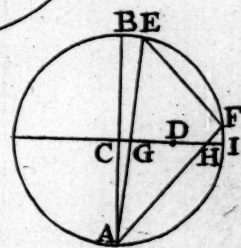


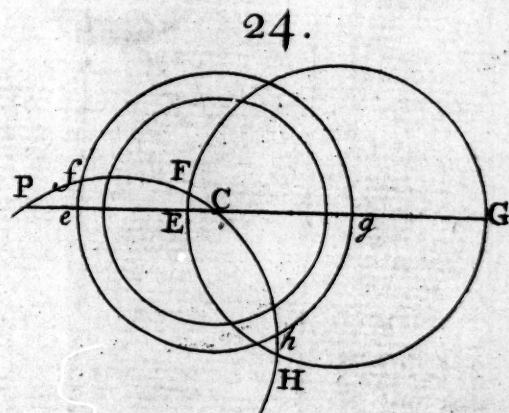
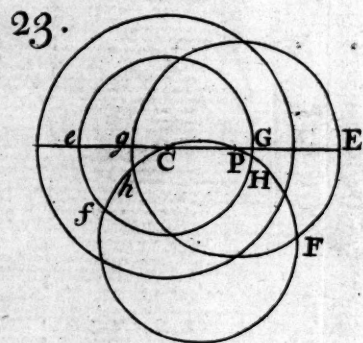
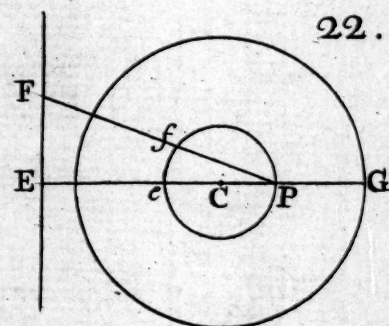
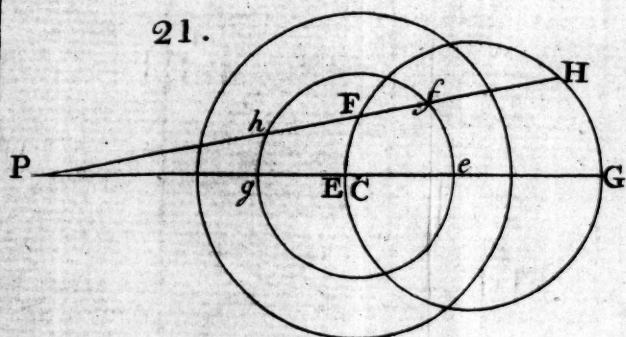
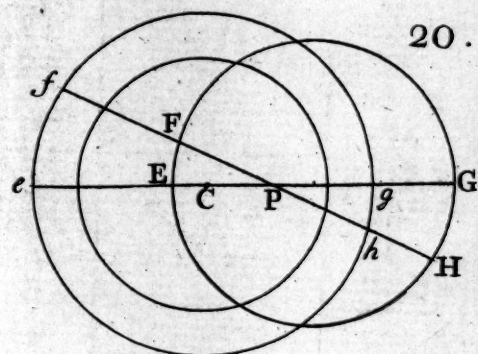
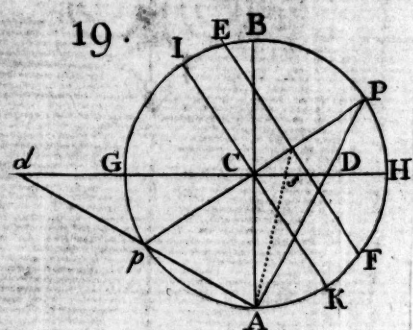
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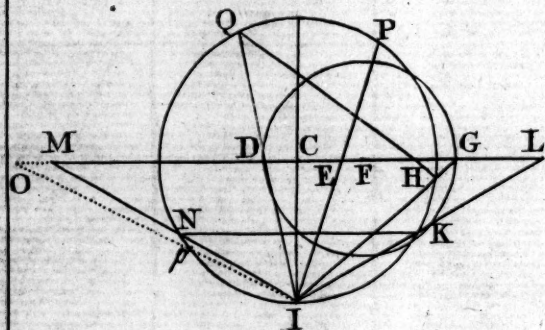


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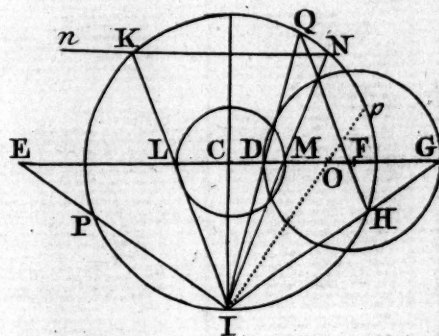




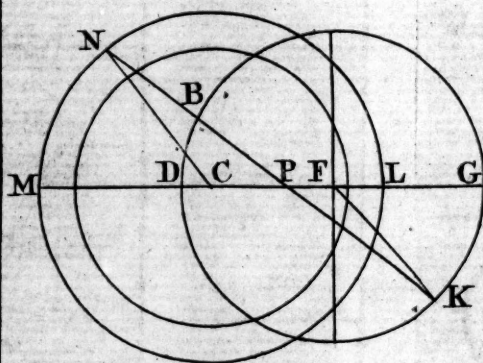
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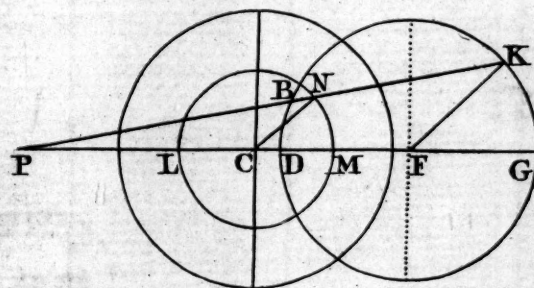
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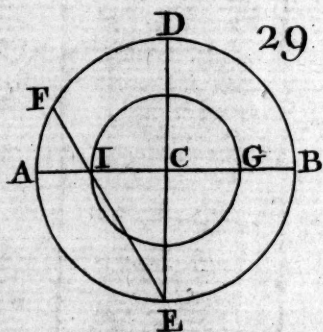
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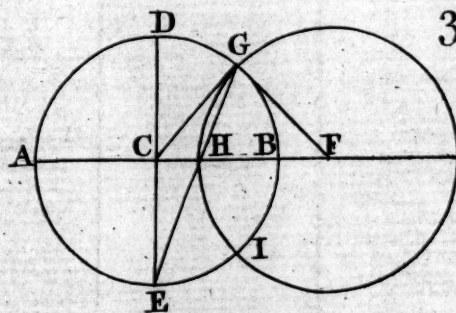
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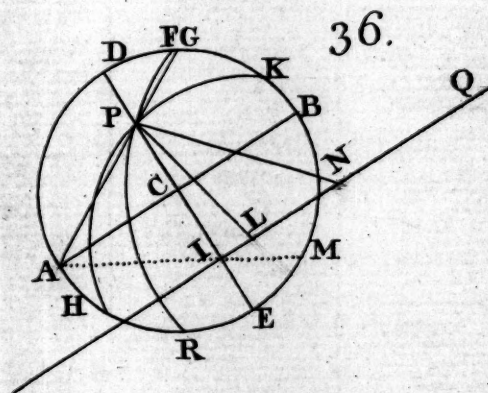
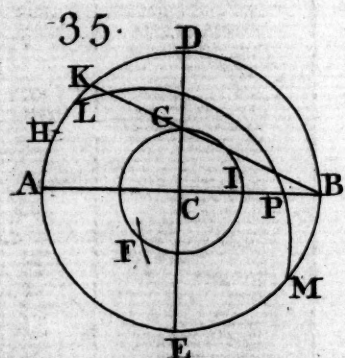
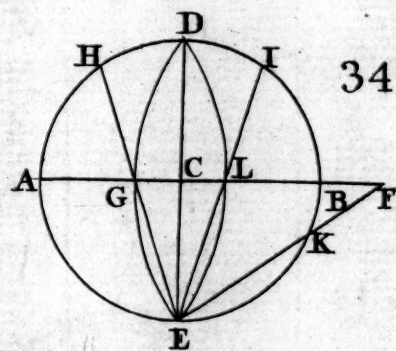
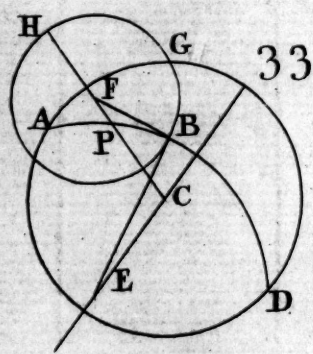
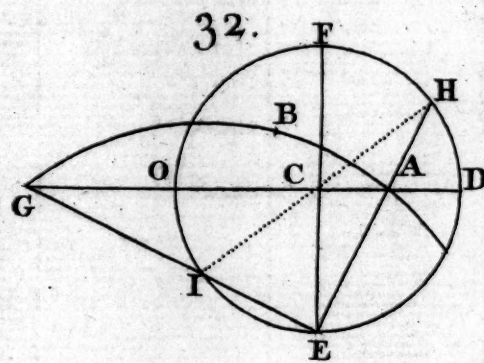
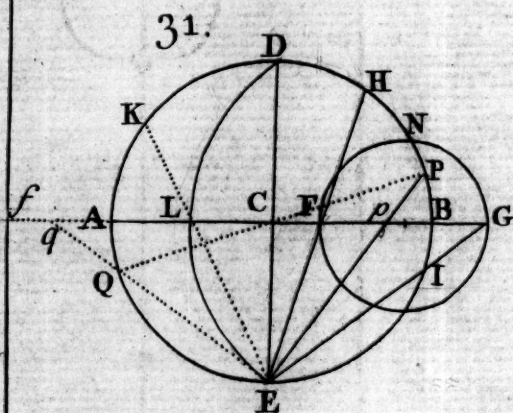


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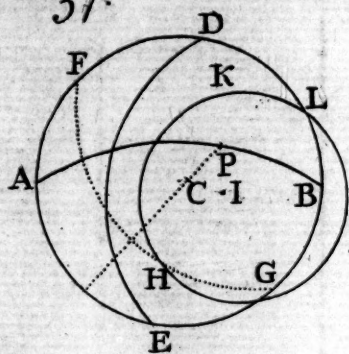


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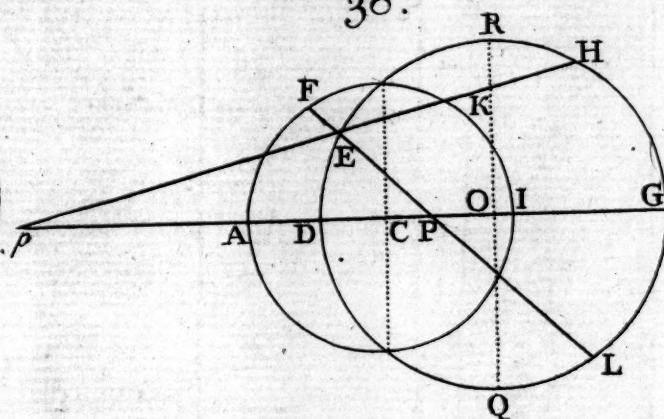




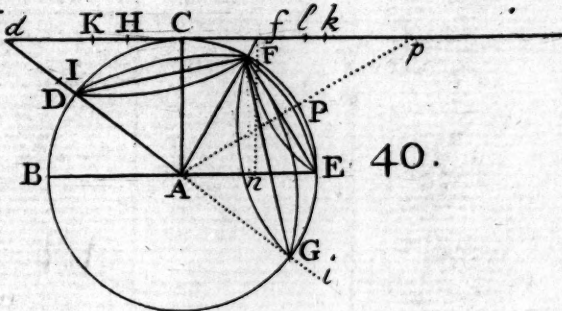
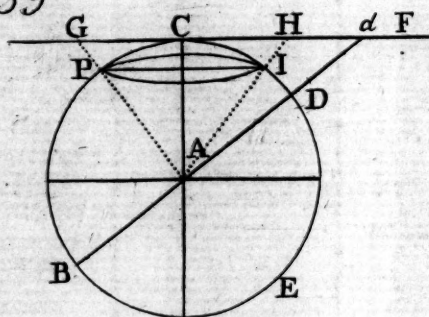
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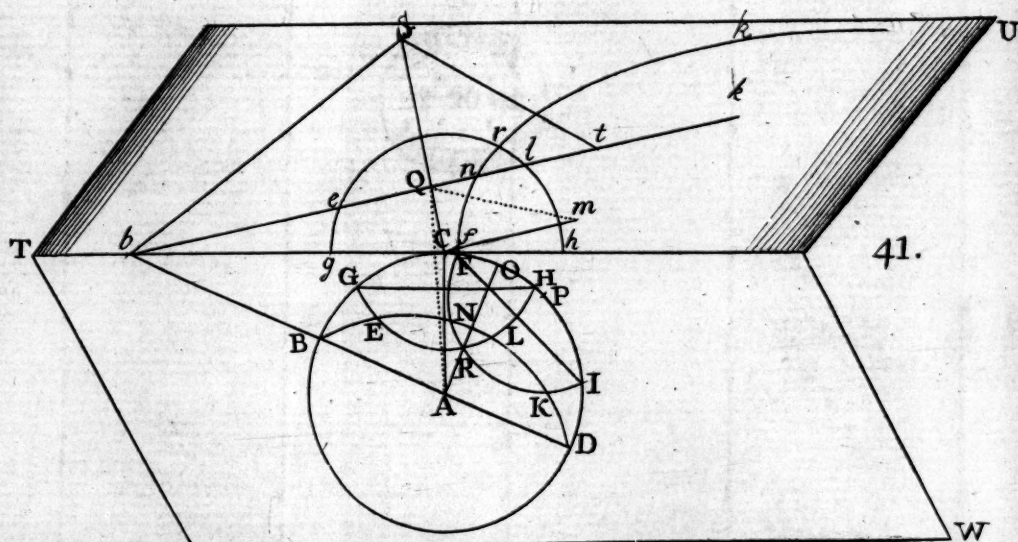
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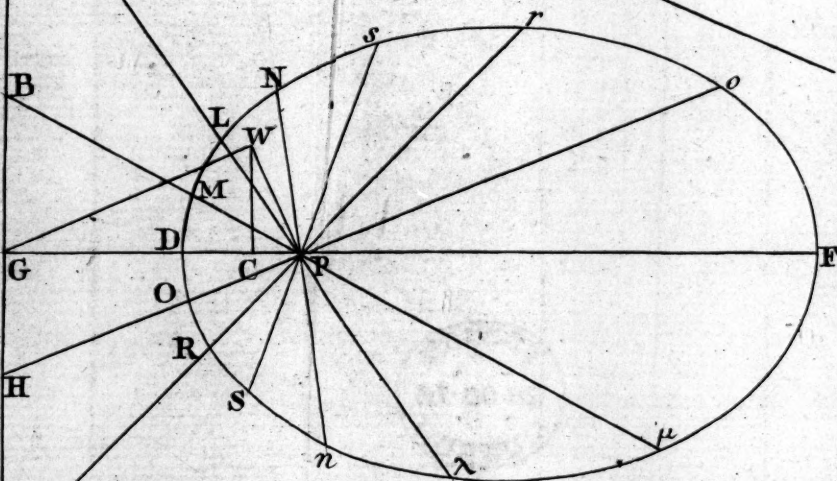
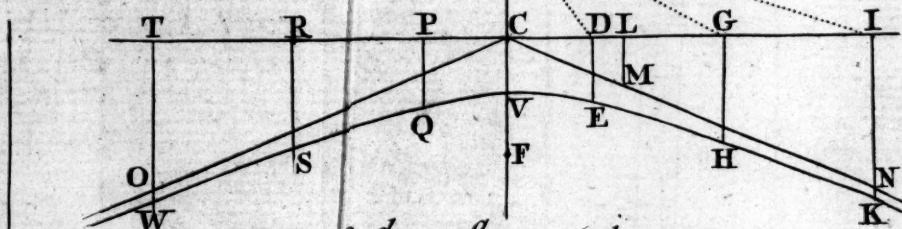


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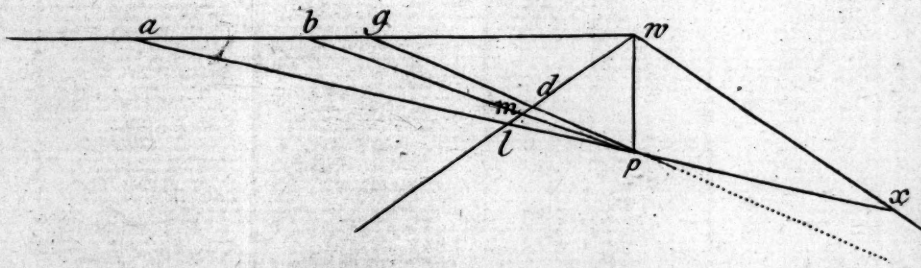


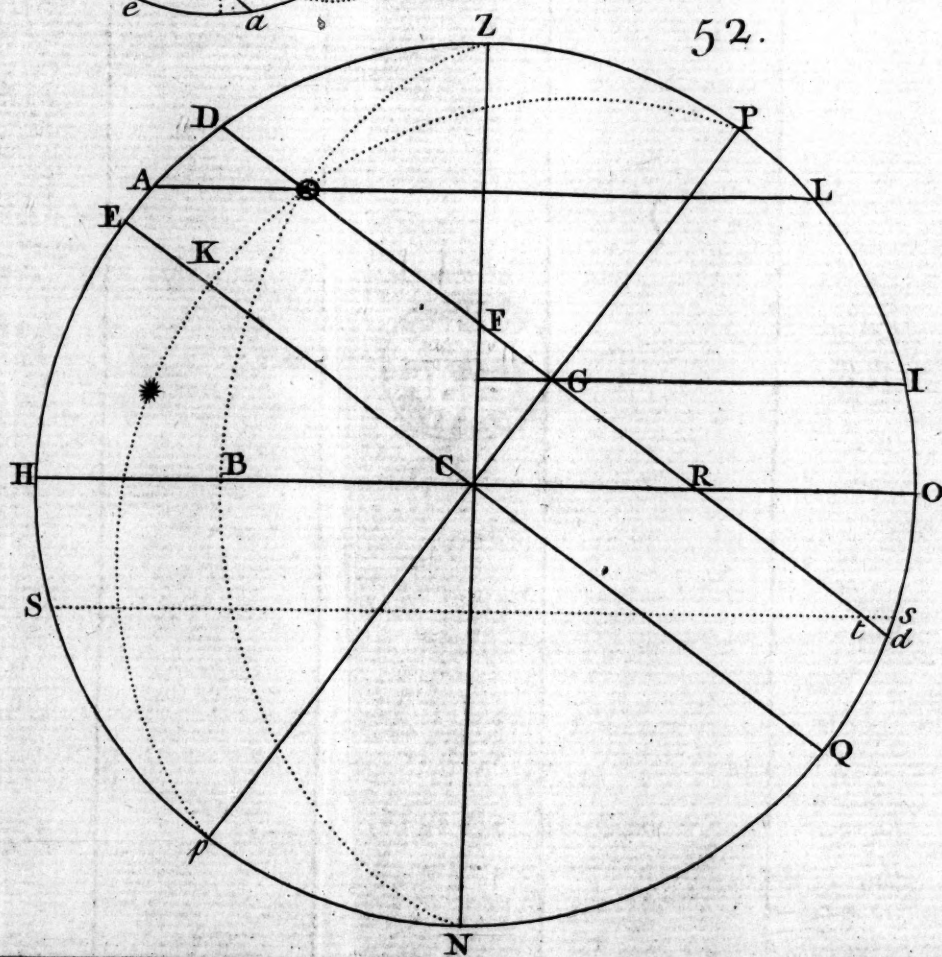
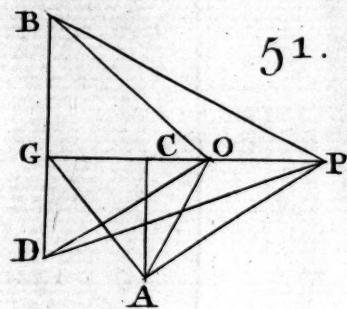
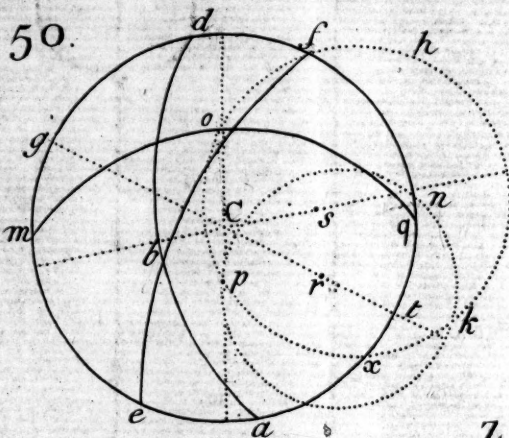
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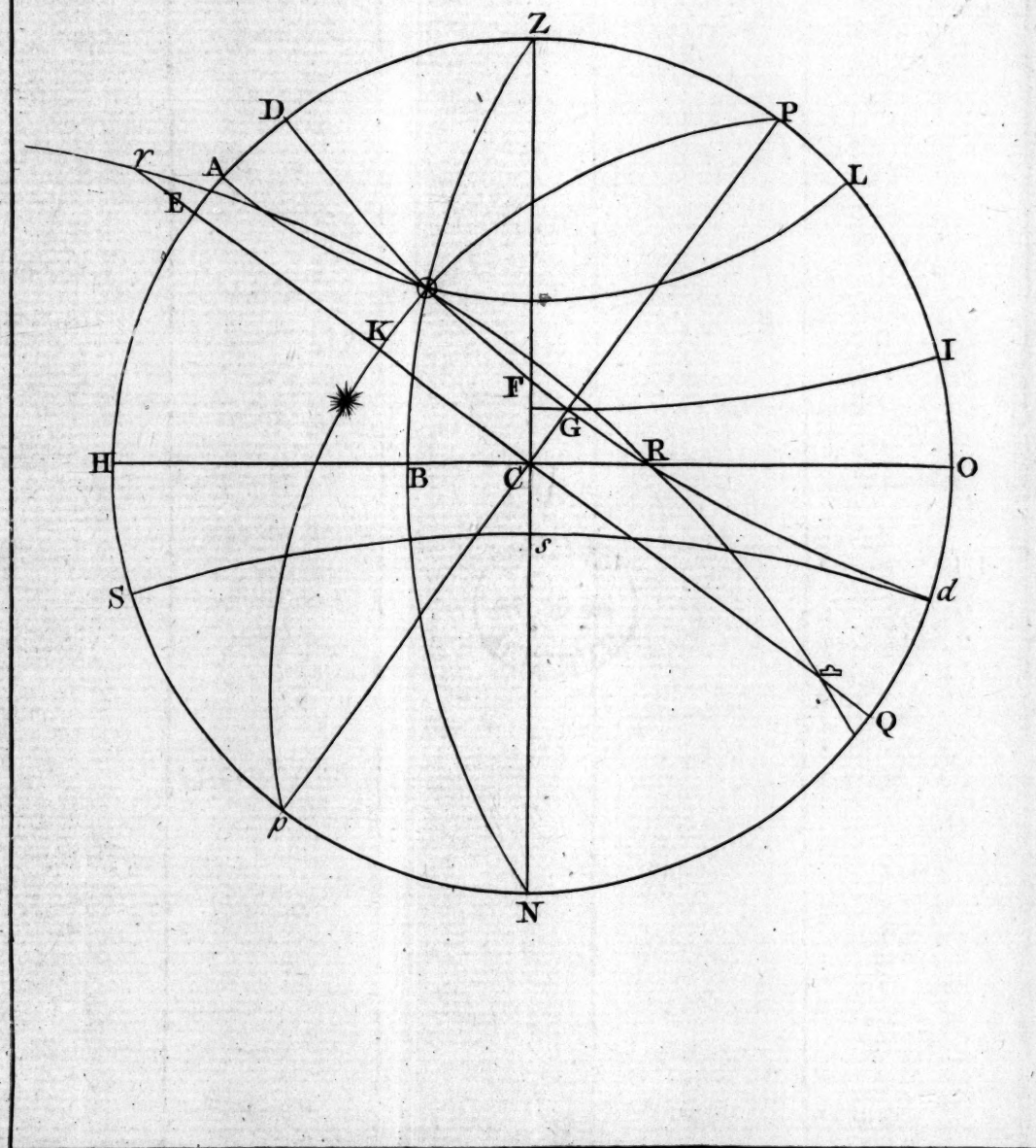
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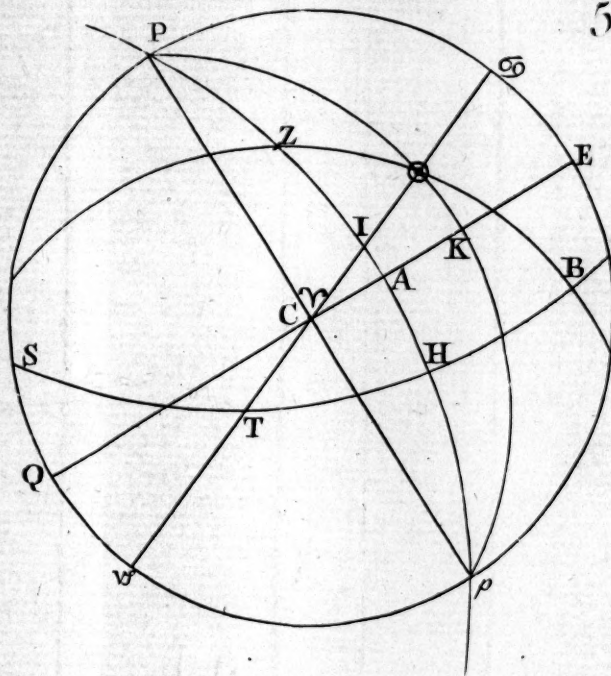




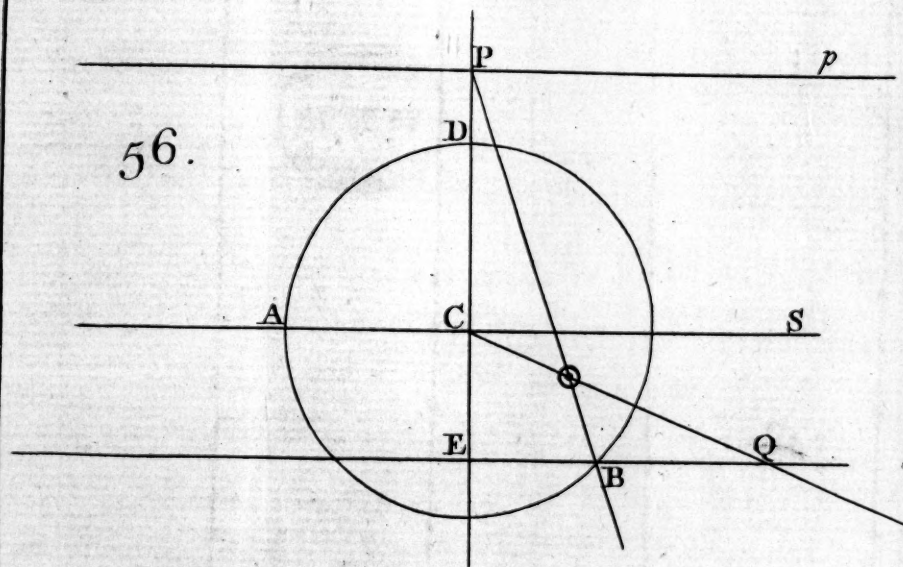


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55.



56.



XII. p. last.